Chapter 13

Universal Gravitation

▲ *An understanding of the law of universal gravitation has allowed scientists to send spacecraft on impressively accurate journeys to other parts of our solar system. This photo of a volcano on Io, a moon of Jupiter, was taken by the Galileo spacecraft, which has been orbiting Jupiter since 1995. The red material has been vented from below the surface. (Univ. of Arizona/JPL/NASA)*

CHAPTER OUTLINE

Before 1687, a large amount of data had been collected on the motions of the Moon and the planets, but a clear understanding of the forces related to these motions was not available. In that year, Isaac Newton provided the key that unlocked the secrets of the heavens. He knew, from his first law, that a net force had to be acting on the Moon because without such a force the Moon would move in a straight-line path rather than in its almost circular orbit. Newton reasoned that this force was the gravitational attraction exerted by the Earth on the Moon. He realized that the forces involved in the Earth–Moon attraction and in the Sun–planet attraction were not something special to those systems, but rather were particular cases of a general and universal attraction between objects. In other words, Newton saw that the same force of attraction that causes the Moon to follow its path around the Earth also causes an apple to fall from a tree. As he put it, "I deduced that the forces which keep the planets in their orbs must be reciprocally as the squares of their distances from the centers about which they revolve; and thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the Earth; and found them answer pretty nearly."

In this chapter we study the law of universal gravitation. We emphasize a description of planetary motion because astronomical data provide an important test of this law's validity. We then show that the laws of planetary motion developed by Johannes Kepler follow from the law of universal gravitation and the concept of conservation of angular momentum. We conclude by deriving a general expression for gravitational potential energy and examining the energetics of planetary and satellite motion.

13.1 Newton's Law of Universal Gravitation

You may have heard the legend that Newton was struck on the head by a falling apple while napping under a tree. This alleged accident supposedly prompted him to imagine that perhaps all objects in the Universe were attracted to each other in the same way the apple was attracted to the Earth. Newton analyzed astronomical data on the motion of the Moon around the Earth. From that analysis, he made the bold assertion that the force law governing the motion of planets was the *same* as the force law that attracted a falling apple to the Earth. This was the first time that "earthly" and "heavenly" motions were unified. We shall look at the mathematical details of Newton's analysis in this section.

In 1687 Newton published his work on the law of gravity in his treatise *Mathematical Principles of Natural Philosophy*. **Newton's law of universal gravitation** states that

The law of universal gravitation

> every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

If the particles have masses m_1 and m_2 and are separated by a distance r, the magnitude of this gravitational force is

$$F_g = G\frac{m_1 m_2}{r^2} \tag{13.1}$$

where G is a constant, called the *universal gravitational constant,* that has been measured experimentally. Its value in SI units is

$$G = 6.673 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2 \tag{13.2}$$

The form of the force law given by Equation 13.1 is often referred to as an **inverse-square law** because the magnitude of the force varies as the inverse square of the separation of the particles.[1] We shall see other examples of this type of force law in subsequent chapters. We can express this force in vector form by defining a unit vector $\hat{\mathbf{r}}_{12}$ (Fig. 13.1). Because this unit vector is directed from particle 1 toward particle 2, the force exerted by particle 1 on particle 2 is

$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{12} \tag{13.3}$$

where the negative sign indicates that particle 2 is attracted to particle 1, and hence the force on particle 2 must be directed toward particle 1. By Newton's third law, the force exerted by particle 2 on particle 1, designated $\mathbf{F}_{21}$, is equal in magnitude to $\mathbf{F}_{12}$ and in the opposite direction. That is, these forces form an action–reaction pair, and $\mathbf{F}_{21} = -\mathbf{F}_{12}$.

Several features of Equation 13.3 deserve mention. The gravitational force is a field force that always exists between two particles, regardless of the medium that separates them. Because the force varies as the inverse square of the distance between the particles, it decreases rapidly with increasing separation.

Another important point that we can show from Equation 13.3 is that **the gravitational force exerted by a finite-size, spherically symmetric mass distribution on a particle outside the distribution is the same as if the entire mass of the distribution were concentrated at the center.** For example, the magnitude of the force exerted by the Earth on a particle of mass m near the Earth's surface is

$$F_g = G\frac{M_E m}{R_E^{\,2}} \tag{13.4}$$

where M_E is the Earth's mass and R_E its radius. This force is directed toward the center of the Earth.

In formulating his law of universal gravitation, Newton used the following reasoning, which supports the assumption that the gravitational force is proportional to the inverse square of the separation between the two interacting objects. He compared the acceleration of the Moon in its orbit with the acceleration of an object falling near the Earth's surface, such as the legendary apple (Fig. 13.2). Assuming that both accelerations had the same cause—namely, the gravitational attraction of the Earth—Newton used the inverse-square law to reason that the acceleration of the Moon toward the Earth (centripetal acceleration) should be proportional to $1/r_M^{\,2}$, where r_M is the distance between the centers of the Earth and the Moon. Furthermore, the acceleration of the apple toward the Earth should be proportional to $1/R_a^{\,2}$, where R_a is the distance between the centers of the Earth and the apple. Because the apple is located at the surface of the earth, $R_a = R_E$, the radius of the Earth. Using the values $r_M = 3.84 \times 10^8$ m and $R_E = 6.37 \times 10^6$ m, Newton predicted that the ratio of the Moon's acceleration a_M to the apple's acceleration g would be

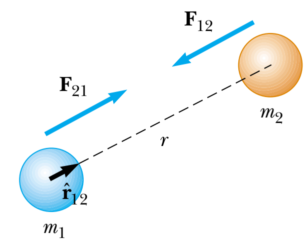
$$\frac{a_M}{g} = \frac{(1/r_M)^2}{(1/R_E)^2} = \left(\frac{R_E}{r_M}\right)^2 = \left(\frac{6.37 \times 10^6\ \text{m}}{3.84 \times 10^8\ \text{m}}\right)^2 = 2.75 \times 10^{-4}$$

[1] An *inverse* proportionality between two quantities x and y is one in which $y = k/x$, where k is a constant. A *direct* proportion between x and y exists when $y = kx$.

PITFALL PREVENTION

13.1 Be Clear on *g* and *G*

The symbol g represents the magnitude of the free-fall acceleration near a planet. At the surface of the Earth, g has the value 9.80 m/s^2. On the other hand, G is a universal constant that has the same value everywhere in the Universe.



Active Figure 13.1 The gravitational force between two particles is attractive. The unit vector $\hat{\mathbf{r}}_{12}$ is directed from particle 1 toward particle 2. Note that $\mathbf{F}_{21} = -\mathbf{F}_{12}$.

At the Active Figures link at http://www/pse6.com, you can change the masses of the particles and the separation distance between the particles to see the effect on the gravitational force.

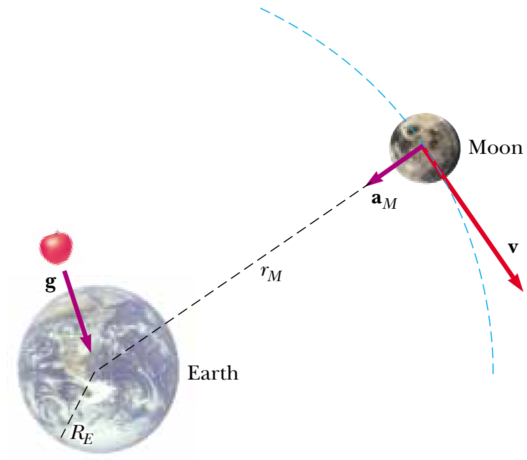


Figure 13.2 As it revolves around the Earth, the Moon experiences a centripetal acceleration $\mathbf{a}_M$ directed toward the Earth. An object near the Earth's surface, such as the apple shown here, experiences an acceleration $\mathbf{g}$. (Dimensions are not to scale.)

Therefore, the centripetal acceleration of the Moon is

$$a_M = (2.75 \times 10^{-4})(9.80 \text{ m/s}^2) = 2.70 \times 10^{-3} \text{ m/s}^2$$

Newton also calculated the centripetal acceleration of the Moon from a knowledge of its mean distance from the Earth and the known value of its orbital period, $T = 27.32$ days $= 2.36 \times 10^6$ s. In a time interval T, the Moon travels a distance $2\pi r_M$, which equals the circumference of its orbit. Therefore, its orbital speed is $2\pi r_M/T$ and its centripetal acceleration is

$$a_M = \frac{v^2}{r_M} = \frac{(2\pi r_M/T)^2}{r_M} = \frac{4\pi^2 r_M}{T^2} = \frac{4\pi^2(3.84 \times 10^8 \text{ m})}{(2.36 \times 10^6 \text{ s})^2}$$
$$= 2.72 \times 10^{-3} \text{ m/s}^2$$

The nearly perfect agreement between this value and the value Newton obtained using g provides strong evidence of the inverse-square nature of the gravitational force law.

Although these results must have been very encouraging to Newton, he was deeply troubled by an assumption he made in the analysis. To evaluate the acceleration of an object at the Earth's surface, Newton treated the Earth as if its mass were all concentrated at its center. That is, he assumed that the Earth acted as a particle as far as its influence on an exterior object was concerned. Several years later, in 1687, on the basis of his pioneering work in the development of calculus, Newton proved that this assumption was valid and was a natural consequence of the law of universal gravitation.

We have evidence that the gravitational force acting on an object is directly proportional to its mass from our observations of falling objects, discussed in Chapter 2. All objects, regardless of mass, fall in the absence of air resistance at the same acceleration g near the surface of the Earth. According to Newton's second law, this acceleration is given by $g = F_g/m$, where m is the mass of the falling object. If this ratio is to be the same for all falling objects, then F_g must be directly proportional to m, so that the mass cancels in the ratio. If we consider the more general situation of a gravitational force between any two objects with mass, such as two planets, this same argument can be applied to show that the gravitational force is proportional to one of the masses. We can choose *either* of the masses in the argument, however; thus, the gravitational force must be directly proportional to *both* masses, as can be seen in Equation 13.3.

Quick Quiz 13.1 The Moon remains in its orbit around the Earth rather than falling to the Earth because (a) it is outside of the gravitational influence of the Earth (b) it is in balance with the gravitational forces from the Sun and other planets (c) the net force on the Moon is zero (d) none of these (e) all of these.

Quick Quiz 13.2 A planet has two moons of equal mass. Moon 1 is in a circular orbit of radius r. Moon 2 is in a circular orbit of radius $2r$. The magnitude of the gravitational force exerted by the planet on moon 2 is (a) four times as large as that on moon 1 (b) twice as large as that on moon 1 (c) equal to that on moon 1 (d) half as large as that on moon 1 (e) one fourth as large as that on moon 1.

Example 13.1 Billiards, Anyone? Interactive

Three 0.300-kg billiard balls are placed on a table at the corners of a right triangle, as shown in Figure 13.3. Calculate the gravitational force on the cue ball (designated m_1) resulting from the other two balls.

Solution First we calculate separately the individual forces on the cue ball due to the other two balls, and then we find the vector sum to obtain the resultant force. We can see graphically that this force should point upward and toward the right. We locate our coordinate axes as shown in Figure 13.3, placing our origin at the position of the cue ball.

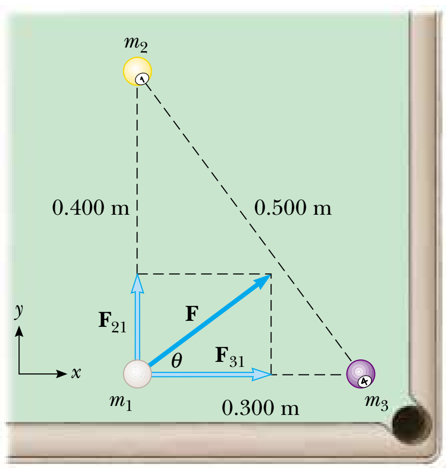


Figure 13.3 (Example 13.1) The resultant gravitational force acting on the cue ball is the vector sum $\mathbf{F}_{21} + \mathbf{F}_{31}$.

The force exerted by m_2 on the cue ball is directed upward and is given by

$$\mathbf{F}_{21} = G\frac{m_2 m_1}{r_{21}^2}\,\hat{\mathbf{j}}$$

$$= (6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(0.300\ \text{kg})(0.300\ \text{kg})}{(0.400\ \text{m})^2}\,\hat{\mathbf{j}}$$

$$= 3.75 \times 10^{-11}\,\hat{\mathbf{j}}\ \text{N}$$

This result shows that the gravitational forces between everyday objects have extremely small magnitudes. The force exerted by m_3 on the cue ball is directed to the right:

$$\mathbf{F}_{31} = G\frac{m_3 m_1}{r_{31}^2}\,\hat{\mathbf{i}}$$

$$= (6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(0.300\ \text{kg})(0.300\ \text{kg})}{(0.300\ \text{m})^2}\,\hat{\mathbf{i}}$$

$$= 6.67 \times 10^{-11}\,\hat{\mathbf{i}}\ \text{N}$$

Therefore, the net gravitational force on the cue ball is

$$\mathbf{F} = \mathbf{F}_{21} + \mathbf{F}_{31} = \boxed{(6.67\hat{\mathbf{i}} + 3.75\hat{\mathbf{j}}) \times 10^{-11}\ \text{N}}$$

and the magnitude of this force is

$$F = \sqrt{F_{21}{}^2 + F_{31}{}^2} = \sqrt{(3.75)^2 + (6.67)^2} \times 10^{-11}\ \text{N}$$
$$= 7.65 \times 10^{-11}\ \text{N}$$

From $\tan\theta = 3.75/6.67 = 0.562$, the direction of the net gravitational force is $\theta = 29.3°$ counterclockwise from the x axis.

At the Interactive Worked Example link at **http://www.pse6.com,** *you can move balls 2 and 3 to see the effect on the net gravitational force on ball 1.*

13.2 Measuring the Gravitational Constant

The universal gravitational constant G was measured in an important experiment by Henry Cavendish (1731–1810) in 1798. The Cavendish apparatus consists of two small spheres, each of mass m, fixed to the ends of a light horizontal rod suspended by a fine fiber or thin metal wire, as illustrated in Figure 13.4. When two large spheres, each of mass M, are placed near the smaller ones, the attractive force between smaller and larger spheres causes the rod to rotate and twist the wire suspension to a new equilibrium orientation. The angle of rotation is measured by the deflection of a light beam reflected from a mirror attached to the vertical suspension. The deflection of the light beam is an effective technique for amplifying the motion. The experiment is carefully repeated with different masses at various separations. In addition to providing a value

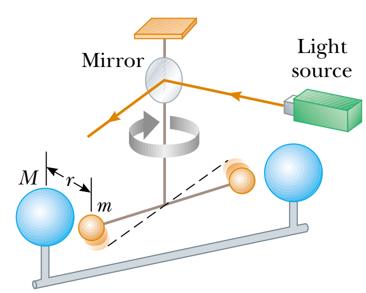


Figure 13.4 Cavendish apparatus for measuring G. The dashed line represents the original position of the rod.

for G, the results show experimentally that the force is attractive, proportional to the product mM, and inversely proportional to the square of the distance r.

13.3 Free-Fall Acceleration and the Gravitational Force

In Chapter 5, when defining mg as the weight of an object of mass m, we referred to g as the magnitude of the free-fall acceleration. Now we are in a position to obtain a more fundamental description of g. Because the magnitude of the force acting on a freely falling object of mass m near the Earth's surface is given by Equation 13.4, we can equate mg to this force to obtain

$$mg = G\frac{M_E m}{{R_E}^2}$$

$$g = G\frac{M_E}{{R_E}^2} \qquad (13.5)$$

Now consider an object of mass m located a distance h above the Earth's surface or a distance r from the Earth's center, where $r = R_E + h$. The magnitude of the gravitational force acting on this object is

$$F_g = G\frac{M_E m}{r^2} = G\frac{M_E m}{(R_E + h)^2}$$

The magnitude of the gravitational force acting on the object at this position is also $F_g = mg$, where g is the value of the free-fall acceleration at the altitude h. Substituting this expression for F_g into the last equation shows that g is

Variation of g with altitude

$$g = \frac{GM_E}{r^2} = \frac{GM_E}{(R_E + h)^2} \qquad (13.6)$$

Thus, it follows that g *decreases* with *increasing altitude*. Because the weight of an object is mg, we see that as $r \to \infty$, its weight approaches zero.

Courtesy NASA

Astronauts F. Story Musgrave and Jeffrey A. Hoffman, along with the Hubble Space Telescope and the space shuttle *Endeavor*, are all in free fall while orbiting the Earth.

Quick Quiz 13.3 Superman stands on top of a very tall mountain and throws a baseball horizontally with a speed such that the baseball goes into a circular orbit around the Earth. While the baseball is in orbit, the acceleration of the ball (a) depends on how fast the baseball is thrown (b) is zero because the ball does not fall to the ground (c) is slightly less than $9.80\ \mathrm{m/s^2}$ (d) is equal to $9.80\ \mathrm{m/s^2}$.

Example 13.2 Variation of *g* with Altitude *h*

The International Space Station operates at an altitude of 350 km. When final construction is completed, it will have a weight (measured at the Earth's surface) of 4.22×10^6 N. What is its weight when in orbit?

Solution We first find the mass of the space station from its weight at the surface of the Earth:

$$m = \frac{F_g}{g} = \frac{4.22 \times 10^6\ \mathrm{N}}{9.80\ \mathrm{m/s^2}} = 4.31 \times 10^5\ \mathrm{kg}$$

This mass is fixed—it is independent of the location of the space station. Because the station is above the surface of the Earth, however, we expect its weight in orbit to be less than its weight on the Earth. Using Equation 13.6 with $h = 350$ km, we obtain

$$\begin{aligned} g &= \frac{GM_E}{(R_E + h)^2} \\ &= \frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.98 \times 10^{24}\ \mathrm{kg})}{(6.37 \times 10^6\ \mathrm{m} + 0.350 \times 10^6\ \mathrm{m})^2} \\ &= 8.83\ \mathrm{m/s^2} \end{aligned}$$

Because this value is about 90% of the value of g at the Earth surface, we expect that the weight of the station at an altitude of 350 km is 90% of the value at the Earth's surface.

Using the value of g at the location of the station, the station's weight in orbit is

$$mg = (4.31 \times 10^5\ \mathrm{kg})(8.83\ \mathrm{m/s^2}) = \boxed{3.80 \times 10^6\ \mathrm{N}}$$

Values of g at other altitudes are listed in Table 13.1.

Table 13.1

Free-Fall Acceleration g at Various Altitudes Above the Earth's Surface

Altitude h (km)	g ($\mathrm{m/s^2}$)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13
∞	0

Example 13.3 The Density of the Earth

Using the known radius of the Earth and the fact that $g = 9.80\ \mathrm{m/s^2}$ at the Earth's surface, find the average density of the Earth.

Solution From Eq. 1.1, we know that the average density is

$$\rho = \frac{M_E}{V_E}$$

where M_E is the mass of the Earth and V_E is its volume.

From Equation 13.5, we can relate the mass of the Earth to the value of g:

$$g = G\frac{M_E}{{R_E}^2} \longrightarrow M_E = \frac{g{R_E}^2}{G}$$

Substituting this into the definition of density, we obtain

$$\begin{aligned} \rho_E &= \frac{M_E}{V_E} = \frac{(g{R_E}^2/G)}{\frac{4}{3}\pi {R_E}^3} = \frac{3}{4}\frac{g}{\pi G R_E} \\ &= \frac{3}{4}\frac{9.80\ \mathrm{m/s^2}}{\pi(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(6.37 \times 10^6\ \mathrm{m})} \\ &= \boxed{5.51 \times 10^3\ \mathrm{kg/m^3}} \end{aligned}$$

What If? **What if you were told that a typical density of granite at the Earth's surface were $2.75 \times 10^3\ \mathrm{kg/m^3}$—what would you conclude about the density of the material in the Earth's interior?**

Answer Because this value is about half the density that we calculated as an average for the entire Earth, we conclude that the inner core of the Earth has a density much higher than the average value. It is most amazing that the Cavendish experiment, which determines G and can be done on a tabletop, combined with simple free-fall measurements of g provides information about the core of the Earth!

Johannes Kepler
German astronomer (1571–1630)

The German astronomer Kepler is best known for developing the laws of planetary motion based on the careful observations of Tycho Brahe. *(Art Resource)*

13.4 Kepler's Laws and the Motion of Planets

People have observed the movements of the planets, stars, and other celestial objects for thousands of years. In early history, scientists regarded the Earth as the center of the Universe. This so-called *geocentric model* was elaborated and formalized by the Greek astronomer Claudius Ptolemy (c. 100–c. 170) in the second century A.D. and was accepted for the next 1 400 years. In 1543 the Polish astronomer Nicolaus Copernicus (1473–1543) suggested that the Earth and the other planets revolved in circular orbits around the Sun (the *heliocentric model*).

The Danish astronomer Tycho Brahe (1546–1601) wanted to determine how the heavens were constructed, and thus he developed a program to determine the positions of both stars and planets. It is interesting to note that those observations of the planets and 777 stars visible to the naked eye were carried out with only a large sextant and a compass. (The telescope had not yet been invented.)

The German astronomer Johannes Kepler was Brahe's assistant for a short while before Brahe's death, whereupon he acquired his mentor's astronomical data and spent 16 years trying to deduce a mathematical model for the motion of the planets. Such data are difficult to sort out because the Earth is also in motion around the Sun. After many laborious calculations, Kepler found that Brahe's data on the revolution of Mars around the Sun provided the answer.

Kepler's complete analysis of planetary motion is summarized in three statements known as **Kepler's laws:**

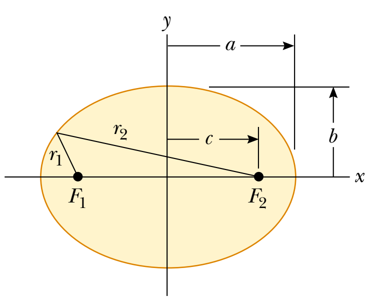
Kepler's laws

1. All planets move in elliptical orbits with the Sun at one focus.
2. The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals.
3. The square of the orbital period of any planet is proportional to the cube of the semimajor axis of the elliptical orbit.

We discuss each of these laws below.

Kepler's First Law

We are familiar with circular orbits of objects around gravitational force centers from our discussions in this chapter. Kepler's first law indicates that the circular orbit is a very special case and elliptical orbits are the general situation. This was a difficult notion for scientists of the time to accept, because they felt that perfect circular orbits of the planets reflected the perfection of heaven.



Active Figure 13.5 Plot of an ellipse. The semimajor axis has length a, and the semiminor axis has length b. Each focus is located at a distance c from the center on each side of the center.

At the Active Figures link at http://www.pse6.com, you can move the focal points or enter values for a, b, c, and e to see the resulting elliptical shape.

Figure 13.5 shows the geometry of an ellipse, which serves as our model for the elliptical orbit of a planet. An ellipse is mathematically defined by choosing two points F_1 and F_2, each of which is a called a **focus,** and then drawing a curve through points for which the sum of the distances r_1 and r_2 from F_1 and F_2, respectively, is a constant. The longest distance through the center between points on the ellipse (and passing through both foci) is called the **major axis,** and this distance is $2a$. In Figure 13.5, the major axis is drawn along the x direction. The distance a is called the **semimajor axis.** Similarly, the shortest distance through the center between points on the ellipse is called the **minor axis** of length $2b$, where the distance b is the **semiminor axis.** Either focus of the ellipse is located at a distance c from the center of the ellipse, where $a^2 = b^2 + c^2$. In the elliptical orbit of a planet around the Sun, the Sun is at one focus of the ellipse. There is nothing at the other focus.

The **eccentricity** of an ellipse is defined as $e = c/a$ and describes the general shape of the ellipse. For a circle, $c = 0$, and the eccentricity is therefore zero. The smaller b is than a, the shorter the ellipse is along the y direction compared to its extent in the x direction in Figure 13.5. As b decreases, c increases, and the eccentricity e increases.

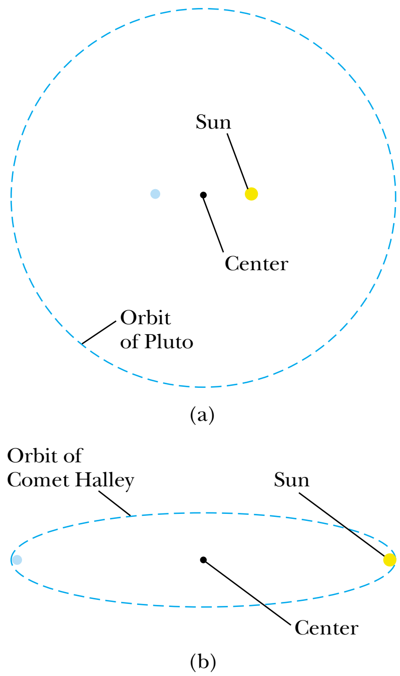


Figure 13.6 (a) The shape of the orbit of Pluto, which has the highest eccentricity ($e = 0.25$) among the planets in the solar system. The Sun is located at the large yellow dot, which is a focus of the ellipse. There is nothing physical located at the center (the small dot) or the other focus (the blue dot). (b) The shape of the orbit of Comet Halley.

PITFALL PREVENTION

13.2 Where is the Sun?

The Sun is located at one focus of the elliptical orbit of a planet. It is *not* located at the center of the ellipse.

Thus, higher values of eccentricity correspond to longer and thinner ellipses. The range of values of the eccentricity for an ellipse is $0 < e < 1$.

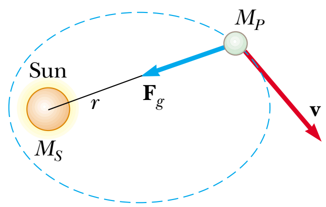
Eccentricities for planetary orbits vary widely in the solar system. The eccentricity of the Earth's orbit is 0.017, which makes it nearly circular. On the other hand, the eccentricity of Pluto's orbit is 0.25, the highest of all the nine planets. Figure 13.6a shows an ellipse with the eccentricity of that of Pluto's orbit. Notice that even this highest-eccentricity orbit is difficult to distinguish from a circle. This is why Kepler's first law is an admirable accomplishment. The eccentricity of the orbit of Comet Halley is 0.97, describing an orbit whose major axis is much longer than its minor axis, as shown in Figure 13.6b. As a result, Comet Halley spends much of its 76-year period far from the Sun and invisible from the Earth. It is only visible to the naked eye during a small part of its orbit when it is near the Sun.

Now imagine a planet in an elliptical orbit such as that shown in Figure 13.5, with the Sun at focus F_2. When the planet is at the far left in the diagram, the distance between the planet and the Sun is $a + c$. This point is called the *aphelion*, where the planet is the farthest away from the Sun that it can be in the orbit. (For an object in orbit around the Earth, this point is called the *apogee*). Conversely, when the planet is at the right end of the ellipse, the point is called the *perihelion* (for an Earth orbit, the *perigee*), and the distance between the planet and the Sun is $a - c$.

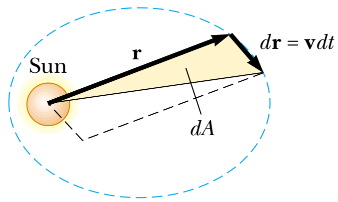
Kepler's first law is a direct result of the inverse square nature of the gravitational force. We have discussed circular and elliptical orbits. These are the allowed shapes of orbits for objects that are *bound* to the gravitational force center. These objects include planets, asteroids, and comets that move repeatedly around the Sun, as well as moons orbiting a planet. There could also be *unbound* objects, such as a meteoroid from deep space that might pass by the Sun once and then never return. The gravitational force between the Sun and these objects also varies as the inverse square of the separation distance, and the allowed paths for these objects include parabolas ($e = 1$) and hyperbolas ($e > 1$).

Kepler's Second Law

Kepler's second law can be shown to be a consequence of angular momentum conservation as follows. Consider a planet of mass M_P moving about the Sun in an elliptical orbit (Fig. 13.7a). Let us consider the planet as a system. We will model the Sun to be



(a)



(b)

Active Figure 13.7 (a) The gravitational force acting on a planet is directed toward the Sun. (b) As a planet orbits the Sun, the area swept out by the radius vector in a time interval dt is equal to half the area of the parallelogram formed by the vectors $\mathbf{r}$ and $d\mathbf{r} = \mathbf{v}\,dt$.

At the Active Figures link at http://www.pse6.com, you can assign a value of the eccentricity and see the resulting motion of the planet around the Sun.

so much more massive than the planet that the Sun does not move. The gravitational force acting on the planet is a central force, always along the radius vector, directed toward the Sun (Fig. 13.7a). The torque on the planet due to this central force is clearly zero, because $\mathbf{F}$ is parallel to $\mathbf{r}$. That is

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times F(r)\hat{\mathbf{r}} = 0$$

Recall that the external net torque on a system equals the time rate of change of angular momentum of the system; that is, $\boldsymbol{\tau} = d\mathbf{L}/dt$. Therefore, because $\boldsymbol{\tau} = 0$, **the angular momentum L of the planet is a constant of the motion:**

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = M_P\mathbf{r} \times \mathbf{v} = \text{constant}$$

We can relate this result to the following geometric consideration. In a time interval dt, the radius vector $\mathbf{r}$ in Figure 13.7b sweeps out the area dA, which equals half the area $|\mathbf{r} \times d\mathbf{r}|$ of the parallelogram formed by the vectors $\mathbf{r}$ and $d\mathbf{r}$. Because the displacement of the planet in the time interval dt is given by $d\mathbf{r} = \mathbf{v}\,dt$, we have

$$dA = \tfrac{1}{2}|\mathbf{r} \times d\mathbf{r}| = \tfrac{1}{2}|\mathbf{r} \times \mathbf{v}\,dt| = \frac{L}{2M_P}\,dt$$

$$\frac{dA}{dt} = \frac{L}{2M_P} = \text{constant} \tag{13.7}$$

where L and M_P are both constants. Thus, we conclude that **the radius vector from the Sun to any planet sweeps out equal areas in equal times.**

It is important to recognize that this result is a consequence of the fact that the gravitational force is a central force, which in turn implies that angular momentum of the planet is constant. Therefore, the law applies to *any* situation that involves a central force, whether inverse-square or not.

Kepler's Third Law

It is informative to show that Kepler's third law can be predicted from the inverse-square law for circular orbits.[2] Consider a planet of mass M_P that is assumed to be moving about the Sun (mass M_S) in a circular orbit, as in Figure 13.8. Because the gravitational force provides the centripetal acceleration of the planet as it moves in a circle, we use Newton's second law for a particle in uniform circular motion,

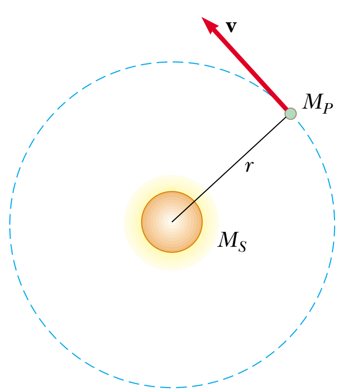


Figure 13.8 A planet of mass M_P moving in a circular orbit around the Sun. The orbits of all planets except Mercury and Pluto are nearly circular.

$$\frac{GM_SM_P}{r^2} = \frac{M_Pv^2}{r}$$

The orbital speed of the planet is $2\pi r/T$, where T is the period; therefore, the preceding expression becomes

$$\frac{GM_S}{r^2} = \frac{(2\pi r/T)^2}{r}$$

$$T^2 = \left(\frac{4\pi^2}{GM_S}\right)r^3 = K_Sr^3$$

where K_S is a constant given by

$$K_S = \frac{4\pi^2}{GM_S} = 2.97 \times 10^{-19}\ \text{s}^2/\text{m}^3$$

[2] The orbits of all planets except Mercury and Pluto are very close to being circular; hence, we do not introduce much error with this assumption. For example, the ratio of the semiminor axis to the semimajor axis for the Earth's orbit is $b/a = 0.999\,86$.

Table 13.2

Useful Planetary Data

Body	Mass (kg)	Mean Radius (m)	Period of Revolution (s)	Mean Distance from Sun (m)	$\frac{T^2}{r^3}$ (s^2/m^3)
Mercury	3.18×10^{23}	2.43×10^{6}	7.60×10^{6}	5.79×10^{10}	2.97×10^{-19}
Venus	4.88×10^{24}	6.06×10^{6}	1.94×10^{7}	1.08×10^{11}	2.99×10^{-19}
Earth	5.98×10^{24}	6.37×10^{6}	3.156×10^{7}	1.496×10^{11}	2.97×10^{-19}
Mars	6.42×10^{23}	3.37×10^{6}	5.94×10^{7}	2.28×10^{11}	2.98×10^{-19}
Jupiter	1.90×10^{27}	6.99×10^{7}	3.74×10^{8}	7.78×10^{11}	2.97×10^{-19}
Saturn	5.68×10^{26}	5.85×10^{7}	9.35×10^{8}	1.43×10^{12}	2.99×10^{-19}
Uranus	8.68×10^{25}	2.33×10^{7}	2.64×10^{9}	2.87×10^{12}	2.95×10^{-19}
Neptune	1.03×10^{26}	2.21×10^{7}	5.22×10^{9}	4.50×10^{12}	2.99×10^{-19}
Pluto	$\approx 1.4 \times 10^{22}$	$\approx 1.5 \times 10^{6}$	7.82×10^{9}	5.91×10^{12}	2.96×10^{-19}
Moon	7.36×10^{22}	1.74×10^{6}	—	—	—
Sun	1.991×10^{30}	6.96×10^{8}	—	—	—

This equation is also valid for elliptical orbits if we replace r with the length a of the semimajor axis (Fig. 13.5):

$$T^2 = \left(\frac{4\pi^2}{GM_S}\right) a^3 = K_S a^3 \tag{13.8}$$

Kepler's third law

Equation 13.8 is Kepler's third law. Because the semimajor axis of a circular orbit is its radius, Equation 13.8 is valid for both circular and elliptical orbits. Note that the constant of proportionality K_S is independent of the mass of the planet. Equation 13.8 is therefore valid for *any* planet.[3] If we were to consider the orbit of a satellite such as the Moon about the Earth, then the constant would have a different value, with the Sun's mass replaced by the Earth's mass, that is, $K_E = 4\pi^2/GM_E$.

Table 13.2 is a collection of useful planetary data. The last column verifies that the ratio T^2/r^3 is constant. The small variations in the values in this column are due to uncertainties in the data measured for the periods and semimajor axes of the planets.

Recent astronomical work has revealed the existence of a large number of solar system objects beyond the orbit of Neptune. In general, these lie in the *Kuiper belt,* a region that extends from about 30 AU (the orbital radius of Neptune) to 50 AU. (An AU is an *astronomical unit*—the radius of the Earth's orbit.) Current estimates identify at least 70 000 objects in this region with diameters larger than 100 km. The first KBO (Kuiper Belt Object) was discovered in 1992. Since then, many more have been detected and some have been given names, such as Varuna (diameter about 900–1 000 km, discovered in 2000), Ixion (diameter about 900–1 000 km, discovered in 2001), and Quaoar (diameter about 800 km, discovered in 2002).

A subset of about 1 400 KBOs are called "Plutinos" because, like Pluto, they exhibit a resonance phenomenon, orbiting the Sun two times in the same time interval as Neptune revolves three times. Some astronomers even claim that Pluto should not be considered a planet but should be identified as a KBO. The contemporary application of Kepler's laws and such exotic proposals as planetary angular momentum exchange and migrating planets[4] suggest the excitement of this active area of current research.

Quick Quiz 13.4 Pluto, the farthest planet from the Sun, has an orbital period that is (a) greater than a year (b) less than a year (c) equal to a year.

[3] Equation 13.8 is indeed a proportion because the ratio of the two quantities T^2 and a^3 is a constant. The variables in a proportion are not required to be limited to the first power only.

[4] Malhotra, R., "Migrating Planets," *Scientific American,* September 1999, volume 281, number 3.

Quick Quiz 13.5 An asteroid is in a highly eccentric elliptical orbit around the Sun. The period of the asteroid's orbit is 90 days. Which of the following statements is true about the possibility of a collision between this asteroid and the Earth? (a) There is no possible danger of a collision. (b) There is a possibility of a collision. (c) There is not enough information to determine whether there is danger of a collision.

Quick Quiz 13.6 A satellite moves in an elliptical orbit about the Earth such that, at perigee and apogee positions, its distances from the Earth's center are respectively D and $4D$. The relationship between the speeds at these two positions is (a) $v_p = v_a$ (b) $v_p = 4v_a$ (c) $v_a = 4v_p$ (d) $v_p = 2v_a$ (e) $v_a = 2v_p$.

Example 13.4 The Mass of the Sun

Calculate the mass of the Sun using the fact that the period of the Earth's orbit around the Sun is 3.156×10^7 s and its distance from the Sun is 1.496×10^{11} m.

Solution Using Equation 13.8, we find that

$$M_S = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(1.496 \times 10^{11}\text{ m})^3}{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(3.156 \times 10^7\text{ s})^2}$$
$$= \boxed{1.99 \times 10^{30}\text{ kg}}$$

In Example 13.3, an understanding of gravitational forces enabled us to find out something about the density of the Earth's core, and now we have used this understanding to determine the mass of the Sun!

What If? Suppose you were asked for the mass of Mars. How could you determine this value?

Answer Kepler's third law is valid for any system of objects in orbit around an object with a large mass. Mars has two moons, Phobos and Deimos. If we rewrite Equation 13.8 for these moons of Mars, we have

$$T^2 = \left(\frac{4\pi^2}{GM_M}\right) a^3$$

where M_M is the mass of Mars. Solving for this mass,

$$M_M = \left(\frac{4\pi^2}{G}\right)\frac{a^3}{T^2} = \left(\frac{4\pi^2}{6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2}\right)\frac{a^3}{T^2}$$
$$= (5.92 \times 10^{11}\text{ kg}\cdot\text{s}^2/\text{m}^3)\frac{a^3}{T^2}$$

Phobos has an orbital period of 0.32 days and an almost circular orbit of radius 9 380 km. The orbit of Deimos is even more circular, with a radius of 23 460 km and an orbital period of 1.26 days. Let us calculate the mass of Mars using each of these sets of data:

Phobos:

$$M_M = (5.92 \times 10^{11}\text{ kg}\cdot\text{s}^2/\text{m}^3) \times \frac{(9.380 \times 10^6\text{ m})^3}{(0.32\text{ d})^2}\left(\frac{1\text{ d}}{86\,400\text{ s}}\right)^2 = 6.39 \times 10^{23}\text{ kg}$$

Deimos:

$$M_M = (5.92 \times 10^{11}\text{ kg}\cdot\text{s}^2/\text{m}^3) \times \frac{(2.346 \times 10^7\text{ m})^3}{(1.26\text{ d})^2}\left(\frac{1\text{ d}}{86\,400\text{ s}}\right)^2 = 6.45 \times 10^{23}\text{ kg}$$

These two calculations are within 1% of each other and both are within 0.5% of the value of the mass of Mars given in Table 13.2.

Example 13.5 A Geosynchronous Satellite — Interactive

Consider a satellite of mass m moving in a circular orbit around the Earth at a constant speed v and at an altitude h above the Earth's surface, as illustrated in Figure 13.9.

(A) Determine the speed of the satellite in terms of G, h, R_E (the radius of the Earth), and M_E (the mass of the Earth).

Solution Conceptualize by imagining the satellite moving around the Earth in a circular orbit under the influence of the gravitational force. The satellite must have a centripetal acceleration. Thus, we categorize this problem as one involving Newton's second law, the law of universal gravitation, and circular motion. To analyze the problem, note that the only external force acting on the satellite is the gravitational force, which acts toward the center of the Earth and keeps the satellite in its circular orbit. Therefore, the net force on the satellite is the gravitational force

$$F_r = F_g = G\frac{M_E m}{r^2}$$

From Newton's second law and the fact that the acceleration of the satellite is centripetal, we obtain

$$G\frac{M_E m}{r^2} = m\frac{v^2}{r}$$

Solving for v and remembering that the distance r from the center of the Earth to the satellite is $r = R_E + h$, we obtain

$$(1) \qquad v = \sqrt{\frac{GM_E}{r}} = \boxed{\sqrt{\frac{GM_E}{R_E + h}}}$$

(B) If the satellite is to be *geosynchronous* (that is, appearing to remain over a fixed position on the Earth), how fast is it moving through space?

Solution In order to appear to remain over a fixed position on the Earth, the period of the satellite must be 24 h and the satellite must be in orbit directly over the equator. From Kepler's third law (Equation 13.8) with $a = r$ and $M_S \rightarrow M_E$, we find the radius of the orbit:

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right) r^3$$

$$r = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}}$$

Substituting numerical values and noting that the period is $T = 24\text{ h} = 86\,400\text{ s}$, we find

$$r = \sqrt[3]{\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})(86\,400\text{ s})^2}{4\pi^2}}$$

$$= 4.23 \times 10^7\text{ m}$$

To find the speed of the satellite, we use Equation (1):

$$v = \sqrt{\frac{GM_E}{r}}$$

$$= \sqrt{\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})}{4.23 \times 10^7\text{ m}}}$$

$$= \boxed{3.07 \times 10^3\text{ m/s}}$$

To finalize this problem, it is interesting to note that the value of r calculated here translates to a height of the satellite above the surface of the Earth of almost 36 000 km. Thus, geosynchronous satellites have the advantage of allowing an earthbound antenna to be aimed in a fixed direction, but there is a disadvantage in that the signals between Earth and the satellite must travel a long distance. It is difficult to use geosynchronous satellites for optical observation of the Earth's surface because of their high altitude.

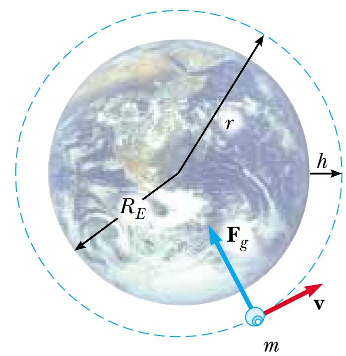


Figure 13.9 (Example 13.5) A satellite of mass m moving around the Earth in a circular orbit of radius r with constant speed v. The only force acting on the satellite is the gravitational force $\mathbf{F}_g$. (Not drawn to scale.)

What If? What if the satellite motion in part (A) were taking place at height *h* above the surface of another planet more massive than the Earth but of the same radius? Would the satellite be moving at a higher or a lower speed than it does around the Earth?

Answer If the planet pulls downward on the satellite with more gravitational force due to its larger mass, the satellite would have to move with a higher speed to avoid moving toward the surface. This is consistent with the predictions of Equation (1), which shows that because the speed v is proportional to the square root of the mass of the planet, as the mass increases, the speed also increases.

You can adjust the altitude of the satellite at the Interactive Worked Example link at **http://www.pse6.com.**

13.5 The Gravitational Field

When Newton published his theory of universal gravitation, it was considered a success because it satisfactorily explained the motion of the planets. Since 1687 the same theory has been used to account for the motions of comets, the deflection of a Cavendish balance, the orbits of binary stars, and the rotation of galaxies. Nevertheless, both Newton's contemporaries and his successors found it difficult to accept the concept of a force that acts at a distance, as mentioned in Section 5.1. They asked how it was possible for two objects to interact when they were not in contact with each other. Newton himself could not answer that question.

An approach to describing interactions between objects that are not in contact came well after Newton's death, and it enables us to look at the gravitational interaction in a different way, using the concept of a **gravitational field** that exists at every point in space. When a particle of mass m is placed at a point where the gravitational

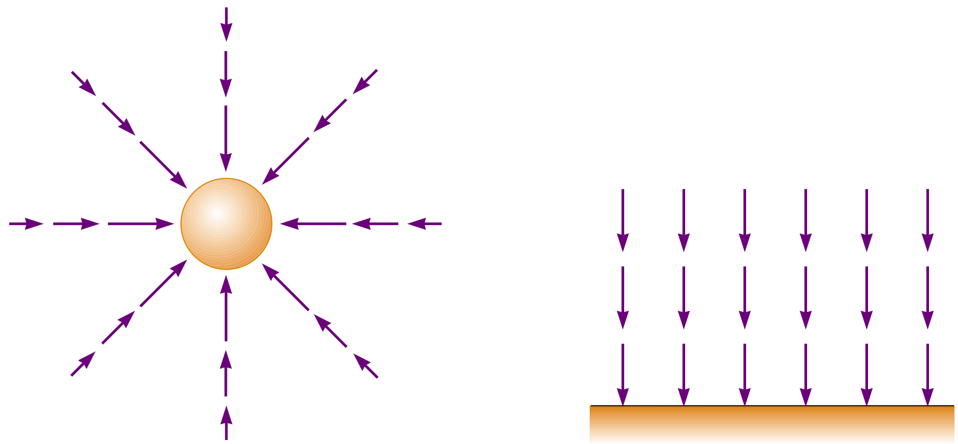


Figure 13.10 (a) The gravitational field vectors in the vicinity of a uniform spherical mass such as the Earth vary in both direction and magnitude. The vectors point in the direction of the acceleration a particle would experience if it were placed in the field. The magnitude of the field vector at any location is the magnitude of the free-fall acceleration at that location. (b) The gravitational field vectors in a small region near the Earth's surface are uniform in both direction and magnitude.

field is $\mathbf{g}$, the particle experiences a force $\mathbf{F}_g = m\mathbf{g}$. In other words, the field exerts a force on the particle. The gravitational field $\mathbf{g}$ is defined as

Gravitational field

$$\mathbf{g} \equiv \frac{\mathbf{F}_g}{m} \tag{13.9}$$

That is, the gravitational field at a point in space equals the gravitational force experienced by a *test particle* placed at that point divided by the mass of the test particle. Notice that the presence of the test particle is not necessary for the field to exist—the Earth creates the gravitational field. We call the object creating the field the *source particle.* (Although the Earth is clearly not a particle, it is possible to show that we can approximate the Earth as a particle for the purpose of finding the gravitational field that it creates.) We can detect the presence of the field and measure its strength by placing a test particle in the field and noting the force exerted on it.

Although the gravitational force is inherently an interaction between two objects, the concept of a gravitational field allows us to "factor out" the mass of one of the objects. In essence, we are describing the "effect" that any object (in this case, the Earth) has on the empty space around itself in terms of the force that *would* be present *if* a second object were somewhere in that space.[5]

As an example of how the field concept works, consider an object of mass m near the Earth's surface. Because the gravitational force acting on the object has a magnitude GM_Em/r^2 (see Eq. 13.4), the field $\mathbf{g}$ at a distance r from the center of the Earth is

$$\mathbf{g} = \frac{\mathbf{F}_g}{m} = -\frac{GM_E}{r^2}\,\hat{\mathbf{r}} \tag{13.10}$$

where $\hat{\mathbf{r}}$ is a unit vector pointing radially outward from the Earth and the negative sign indicates that the field points toward the center of the Earth, as illustrated in Figure 13.10a. Note that the field vectors at different points surrounding the Earth vary in both direction and magnitude. In a small region near the Earth's surface, the downward field $\mathbf{g}$ is approximately constant and uniform, as indicated in Figure 13.10b. Equation 13.10 is valid at all points *outside* the Earth's surface, assuming that the Earth is spherical. At the Earth's surface, where $r = R_E$, $\mathbf{g}$ has a magnitude of 9.80 N/kg. (The unit N/kg is the same as m/s^2.)

[5] We shall return to this idea of mass affecting the space around it when we discuss Einstein's theory of gravitation in Chapter 39.

13.6 Gravitational Potential Energy

In Chapter 8 we introduced the concept of gravitational potential energy, which is the energy associated with the configuration of a system of objects interacting via the gravitational force. We emphasized that the gravitational potential-energy function mgy for a particle–Earth system is valid only when the particle is near the Earth's surface, where the gravitational force is constant. Because the gravitational force between two particles varies as $1/r^2$, we expect that a more general potential-energy function—one that is valid without the restriction of having to be near the Earth's surface—will be significantly different from $U = mgy$.

Before we calculate this general form for the gravitational potential energy function, let us first verify that *the gravitational force is conservative.* (Recall from Section 8.3 that a force is conservative if the work it does on an object moving between any two points is independent of the path taken by the object.) To do this, we first note that the gravitational force is a central force. By definition, a central force is any force that is directed along a radial line to a fixed center and has a magnitude that depends only on the radial coordinate r. Hence, a central force can be represented by $F(r)\hat{\mathbf{r}}$ where $\hat{\mathbf{r}}$ is a unit vector directed from the origin toward the particle, as shown in Figure 13.11.

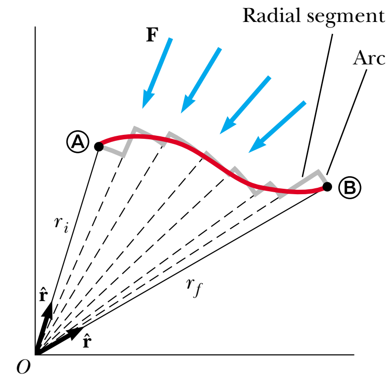


Figure 13.11 A particle moves from Ⓐ to Ⓑ while acted on by a central force **F**, which is directed radially. The path is broken into a series of radial segments and arcs. Because the work done along the arcs is zero, the work done is independent of the path and depends only on r_f and r_i.

Consider a central force acting on a particle moving along the general path Ⓐ to Ⓑ in Figure 13.11. The path from Ⓐ to Ⓑ can be approximated by a series of steps according to the following procedure. In Figure 13.11, we draw several thin wedges, which are shown as dashed lines. The outer boundary of our set of wedges is a path consisting of short radial line segments and arcs (gray in the figure). We select the length of the radial dimension of each wedge such that the short arc at the wedge's wide end intersects the actual path of the particle. Then we can approximate the actual path with a series of zigzag movements that alternate between moving along an arc and moving along a radial line.

By definition, a central force is always directed along one of the radial segments; therefore, the work done by **F** along any radial segment is

$$dW = \mathbf{F} \cdot d\mathbf{r} = F(r)\ dr$$

By definition, the work done by a force that is perpendicular to the displacement is zero. Hence, the work done in moving along any arc is zero because **F** is perpendicular to the displacement along these segments. Therefore, the total work done by **F** is the sum of the contributions along the radial segments:

Work done by a central force

$$W = \int_{r_i}^{r_f} F(r)\ dr$$

where the subscripts i and f refer to the initial and final positions. Because the integrand is a function only of the radial position, this integral depends only on the initial and final values of r. Thus, the work done is the same over *any* path from Ⓐ to Ⓑ. Because the work done is independent of the path and depends only on the end points, we conclude that *any central force is conservative.* We are now assured that a potential energy function can be obtained once the form of the central force is specified.

Recall from Equation 8.15 that the change in the gravitational potential energy of a system associated with a given displacement of a member of the system is defined as the negative of the work done by the gravitational force on that member during the displacement:

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)\ dr \tag{13.11}$$

We can use this result to evaluate the gravitational potential energy function. Consider a particle of mass m moving between two points Ⓐ and Ⓑ above the Earth's surface (Fig. 13.12). The particle is subject to the gravitational force given by Equation 13.1. We can express this force as

$$F(r) = -\frac{GM_E m}{r^2}$$

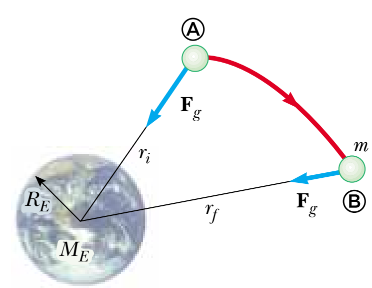


Figure 13.12 As a particle of mass m moves from Ⓐ to Ⓑ above the Earth's surface, the gravitational potential energy changes according to Equation 13.11.

where the negative sign indicates that the force is attractive. Substituting this expression for $F(r)$ into Equation 13.11, we can compute the change in the gravitational potential energy function:

$$U_f - U_i = GM_E m \int_{r_i}^{r_f} \frac{dr}{r^2} = GM_E m \left[-\frac{1}{r} \right]_{r_i}^{r_f}$$

$$U_f - U_i = -GM_E m \left(\frac{1}{r_f} - \frac{1}{r_i} \right) \tag{13.12}$$

As always, the choice of a reference configuration for the potential energy is completely arbitrary. It is customary to choose the reference configuration for zero potential energy to be the same as that for which the force is zero. Taking $U_i = 0$ at $r_i = \infty$, we obtain the important result

Gravitational potential energy of the Earth–particle system for $r > R_E$

$$U(r) = -\frac{GM_E m}{r} \tag{13.13}$$

This expression applies to the Earth–particle system where the particle is separated from the center of the Earth by a distance r, provided that $r \geq R_E$. The result is not valid for particles inside the Earth, where $r < R_E$. Because of our choice of U_i, the function U is always negative (Fig. 13.13).

Although Equation 13.13 was derived for the particle–Earth system, it can be applied to any two particles. That is, the gravitational potential energy associated with any pair of particles of masses m_1 and m_2 separated by a distance r is

$$U = -\frac{Gm_1 m_2}{r} \tag{13.14}$$

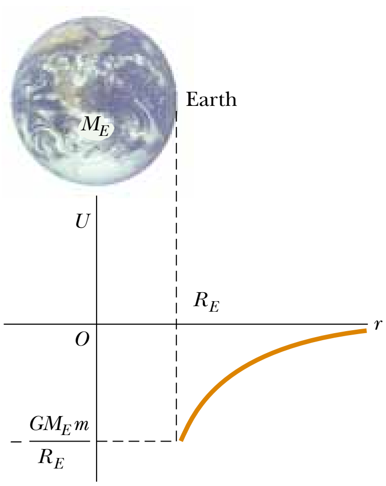


Figure 13.13 Graph of the gravitational potential energy U versus r for an object above the Earth's surface. The potential energy goes to zero as r approaches infinity.

This expression shows that the gravitational potential energy for any pair of particles varies as $1/r$, whereas the force between them varies as $1/r^2$. Furthermore, the potential energy is negative because the force is attractive and we have taken the potential energy as zero when the particle separation is infinite. Because the force between the particles is attractive, we know that an external agent must do positive work to increase the separation between them. The work done by the external agent produces an increase in the potential energy as the two particles are separated. That is, U becomes less negative as r increases.

When two particles are at rest and separated by a distance r, an external agent has to supply an energy at least equal to $+Gm_1m_2/r$ in order to separate the particles to an infinite distance. It is therefore convenient to think of the absolute value of the potential energy as the *binding energy* of the system. If the external agent supplies an energy greater than the binding energy, the excess energy of the system will be in the form of kinetic energy of the particles when the particles are at an infinite separation.

We can extend this concept to three or more particles. In this case, the total potential energy of the system is the sum over all pairs of particles.[6] Each pair contributes a term of the form given by Equation 13.14. For example, if the system contains three particles, as in Figure 13.14, we find that

$$U_{\text{total}} = U_{12} + U_{13} + U_{23} = -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_2 m_3}{r_{23}} \right) \tag{13.15}$$

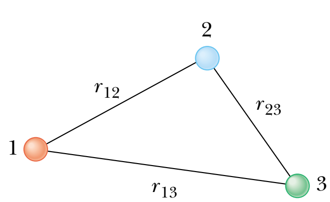


Figure 13.14 Three interacting particles.

The absolute value of U_{total} represents the work needed to separate the particles by an infinite distance.

[6] The fact that potential energy terms can be added for all pairs of particles stems from the experimental fact that gravitational forces obey the superposition principle.

Example 13.6 The Change in Potential Energy

A particle of mass m is displaced through a small vertical distance Δy near the Earth's surface. Show that in this situation the general expression for the change in gravitational potential energy given by Equation 13.12 reduces to the familiar relationship $\Delta U = mg\ \Delta y$.

Solution We can express Equation 13.12 in the form

$$\Delta U = -GM_E m\left(\frac{1}{r_f} - \frac{1}{r_i}\right) = GM_E m\left(\frac{r_f - r_i}{r_i r_f}\right)$$

If both the initial and final positions of the particle are close to the Earth's surface, then $r_f - r_i = \Delta y$ and $r_i r_f \approx R_E{}^2$. (Recall that r is measured from the center of the Earth.) Therefore, the change in potential energy becomes

$$\Delta U \approx \frac{GM_E m}{R_E{}^2}\,\Delta y = mg\ \Delta y$$

where we have used the fact that $g = GM_E/R_E{}^2$ (Eq. 13.5). Keep in mind that the reference configuration is arbitrary because it is the *change* in potential energy that is meaningful.

What If? **Suppose you are performing upper-atmosphere studies and are asked by your supervisor to find the height in the Earth's atmosphere at which the "surface equation" $\Delta U = mg\ \Delta y$ gives a 1.0% error in the change in the potential energy. What is this height?**

Answer Because the surface equation assumes a constant value for g, it will give a ΔU value that is larger than the value given by the general equation, Equation 13.12. Thus, a 1.0% error would be described by the ratio

$$\frac{\Delta U_{\text{surface}}}{\Delta U_{\text{general}}} = 1.010$$

Substituting the expressions for each of these changes ΔU, we have

$$\frac{mg\ \Delta y}{GM_E m(\Delta y/r_i r_f)} = \frac{g r_i r_f}{GM_E} = 1.010$$

where $r_i = R_E$ and $r_f = R_E + \Delta y$. Substituting for g from Equation 13.5, we find

$$\frac{(GM_E/R_E{}^2)R_E(R_E + \Delta y)}{GM_E} = \frac{R_E + \Delta y}{R_E} = 1 + \frac{\Delta y}{R_E} = 1.010$$

Thus,

$$\Delta y = 0.010 R_E = 0.010(6.37 \times 10^6\ \text{m}) = 6.37 \times 10^4\ \text{m} = 63.7\ \text{km}$$

13.7 Energy Considerations in Planetary and Satellite Motion

Consider an object of mass m moving with a speed v in the vicinity of a massive object of mass M, where $M \gg m$. The system might be a planet moving around the Sun, a satellite in orbit around the Earth, or a comet making a one-time flyby of the Sun. If we assume that the object of mass M is at rest in an inertial reference frame, then the total mechanical energy E of the two-object system when the objects are separated by a distance r is the sum of the kinetic energy of the object of mass m and the potential energy of the system, given by Equation 13.14:[7]

$$E = K + U$$

$$E = \tfrac{1}{2}mv^2 - \frac{GMm}{r} \tag{13.16}$$

[7] You might recognize that we have ignored the kinetic energy of the larger body. To see that this simplification is reasonable, consider an object of mass m falling toward the Earth. Because the center of mass of the object–Earth system is effectively stationary, it follows from conservation of momentum that $mv = M_E v_E$. Thus, the Earth acquires a kinetic energy equal to

$$\tfrac{1}{2}M_E v_E{}^2 = \tfrac{1}{2}\frac{m^2}{M_E}v^2 = \frac{m}{M_E}K$$

where K is the kinetic energy of the object. Because $M_E \gg m$, this result shows that the kinetic energy of the Earth is negligible.

This equation shows that E may be positive, negative, or zero, depending on the value of v. However, for a bound system,[8] such as the Earth–Sun system, E is necessarily *less than zero* because we have chosen the convention that $U \rightarrow 0$ as $r \rightarrow \infty$.

We can easily establish that $E < 0$ for the system consisting of an object of mass m moving in a circular orbit about an object of mass $M >> m$ (Fig. 13.15). Newton's second law applied to the object of mass m gives

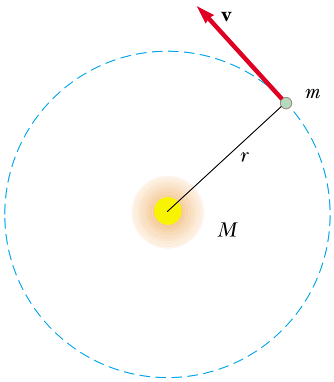


Figure 13.15 An object of mass m moving in a circular orbit about a much larger object of mass M.

$$\frac{GMm}{r^2} = ma = \frac{mv^2}{r}$$

Multiplying both sides by r and dividing by 2 gives

$$\tfrac{1}{2}mv^2 = \frac{GMm}{2r} \qquad (13.17)$$

Substituting this into Equation 13.16, we obtain

$$E = \frac{GMm}{2r} - \frac{GMm}{r}$$

Total energy for circular orbits

$$\boxed{E = -\frac{GMm}{2r}} \qquad \text{(circular orbits)} \qquad (13.18)$$

This result clearly shows that **the total mechanical energy is negative in the case of circular orbits.** Note that **the kinetic energy is positive and equal to half the absolute value of the potential energy.** The absolute value of E is also equal to the binding energy of the system, because this amount of energy must be provided to the system to move the two objects infinitely far apart.

The total mechanical energy is also negative in the case of elliptical orbits. The expression for E for elliptical orbits is the same as Equation 13.18 with r replaced by the semimajor axis length a:

Total energy for elliptical orbits

$$\boxed{E = -\frac{GMm}{2a}} \qquad \text{(elliptical orbits)} \qquad (13.19)$$

Furthermore, the total energy is constant if we assume that the system is isolated. Therefore, as the object of mass m moves from Ⓐ to Ⓑ in Figure 13.12, the total energy remains constant and Equation 13.16 gives

$$E = \tfrac{1}{2}mv_i^2 - \frac{GMm}{r_i} = \tfrac{1}{2}mv_f^2 - \frac{GMm}{r_f} \qquad (13.20)$$

Combining this statement of energy conservation with our earlier discussion of conservation of angular momentum, we see that **both the total energy and the total angular momentum of a gravitationally bound, two-object system are constants of the motion.**

Quick Quiz 13.7 A comet moves in an elliptical orbit around the Sun. Which point in its orbit (perihelion or aphelion) represents the highest value of (a) the speed of the comet (b) the potential energy of the comet–Sun system (c) the kinetic energy of the comet (d) the total energy of the comet–Sun system?

[8] Of the three examples provided at the beginning of this section, the planet moving around the Sun and a satellite in orbit around the Earth are bound systems—the Earth will always stay near the Sun, and the satellite will always stay near the Earth. The one-time comet flyby represents an unbound system—the comet interacts once with the Sun but is not bound to it. Thus, in theory the comet can move infinitely far away from the Sun.

Example 13.7 Changing the Orbit of a Satellite

The space shuttle releases a 470-kg communications satellite while in an orbit 280 km above the surface of the Earth. A rocket engine on the satellite boosts it into a geosynchronous orbit, which is an orbit in which the satellite stays directly over a single location on the Earth. How much energy does the engine have to provide?

Solution We first determine the initial radius (not the altitude above the Earth's surface) of the satellite's orbit when it is still in the shuttle's cargo bay. This is simply

$$R_E + 280 \text{ km} = 6.65 \times 10^6 \text{ m} = r_i$$

In Example 13.5, we found that the radius of the orbit of a geosynchronous satellite is $r_f = 4.23 \times 10^7$ m. Applying Equation 13.18, we obtain, for the total initial and final energies,

$$E_i = -\frac{GM_E m}{2r_i} \qquad E_f = -\frac{GM_E m}{2r_f}$$

The energy required from the engine to boost the satellite is

$$\begin{aligned}\Delta E = E_f - E_i &= -\frac{GM_E m}{2}\left(\frac{1}{r_f} - \frac{1}{r_i}\right) \\ &= -\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(470\ \text{kg})}{2} \\ &\quad \times \left(\frac{1}{4.23 \times 10^7\ \text{m}} - \frac{1}{6.65 \times 10^6\ \text{m}}\right) \\ &= \boxed{1.19 \times 10^{10}\ \text{J}}\end{aligned}$$

This is the energy equivalent of 89 gal of gasoline. NASA engineers must account for the changing mass of the spacecraft as it ejects burned fuel, something we have not done here. Would you expect the calculation that includes the effect of this changing mass to yield a greater or lesser amount of energy required from the engine?

If we wish to determine how the energy is distributed after the engine is fired, we find from Equation 13.17 that the change in kinetic energy is $\Delta K = (GM_E m/2)(1/r_f - 1/r_i) = -1.19 \times 10^{10}$ J (a decrease), and the corresponding change in potential energy is $\Delta U = -GM_E m(1/r_f - 1/r_i) = 2.38 \times 10^{10}$ J (an increase). Thus, the change in orbital energy of the system is $\Delta E = \Delta K + \Delta U = 1.19 \times 10^{10}$ J, as we already calculated. The firing of the engine results in a transformation of chemical potential energy in the fuel to orbital energy of the system. Because an increase in gravitational potential energy is accompanied by a decrease in kinetic energy, we conclude that the speed of an orbiting satellite decreases as its altitude increases.

Escape Speed

Suppose an object of mass m is projected vertically upward from the Earth's surface with an initial speed v_i, as illustrated in Figure 13.16. We can use energy considerations to find the minimum value of the initial speed needed to allow the object to move infinitely far away from the Earth. Equation 13.16 gives the total energy of the system at any point. At the surface of the Earth, $v = v_i$ and $r = r_i = R_E$. When the object reaches its maximum altitude, $v = v_f = 0$ and $r = r_f = r_{\text{max}}$. Because the total energy of the system is constant, substituting these conditions into Equation 13.20 gives

$$\tfrac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{\text{max}}}$$

Solving for v_i^2 gives

$$v_i^2 = 2GM_E\left(\frac{1}{R_E} - \frac{1}{r_{\text{max}}}\right) \qquad (13.21)$$

Therefore, if the initial speed is known, this expression can be used to calculate the maximum altitude h because we know that

$$h = r_{\text{max}} - R_E$$

We are now in a position to calculate **escape speed,** which is the minimum speed the object must have at the Earth's surface in order to approach an infinite separation distance from the Earth. Traveling at this minimum speed, the object continues to

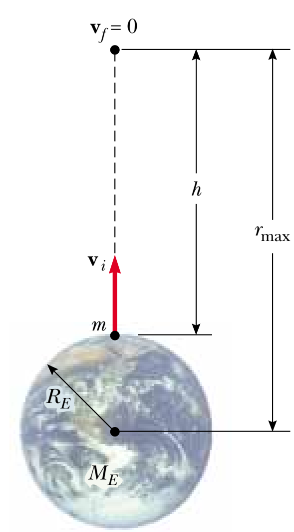


Figure 13.16 An object of mass m projected upward from the Earth's surface with an initial speed v_i reaches a maximum altitude h.

PITFALL PREVENTION

13.3 You Can't Really Escape

Although Equation 13.22 provides the "escape speed" from the Earth, *complete* escape from the Earth's gravitational influence is impossible because the gravitational force is of infinite range. No matter how far away you are, you will always feel some gravitational force due to the Earth.

move farther and farther away from the Earth as its speed asymptotically approaches zero. Letting $r_{max} \rightarrow \infty$ in Equation 13.21 and taking $v_i = v_{esc}$, we obtain

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} \tag{13.22}$$

Note that this expression for v_{esc} is independent of the mass of the object. In other words, a spacecraft has the same escape speed as a molecule. Furthermore, the result is independent of the direction of the velocity and ignores air resistance.

If the object is given an initial speed equal to v_{esc}, the total energy of the system is equal to zero. This can be seen by noting that when $r \rightarrow \infty$, the object's kinetic energy and the potential energy of the system are both zero. If v_i is greater than v_{esc}, the total energy of the system is greater than zero and the object has some residual kinetic energy as $r \rightarrow \infty$.

Example 13.8 Escape Speed of a Rocket

Calculate the escape speed from the Earth for a 5 000-kg spacecraft, and determine the kinetic energy it must have at the Earth's surface in order to move infinitely far away from the Earth.

Solution Using Equation 13.22 gives

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

$$= \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{6.37 \times 10^6\ \text{m}}}$$

$$= \boxed{1.12 \times 10^4\ \text{m/s}}$$

This corresponds to about 25 000 mi/h.

The kinetic energy of the spacecraft is

$$K = \tfrac{1}{2}mv_{esc}^2 = \tfrac{1}{2}(5.00 \times 10^3\ \text{kg})(1.12 \times 10^4\ \text{m/s})^2$$

$$= \boxed{3.14 \times 10^{11}\ \text{J}}$$

This is equivalent to about 2 300 gal of gasoline.

What If? What if we wish to launch a 1 000-kg spacecraft at the escape speed? How much energy does this require?

Answer In Equation 13.22, the mass of the object moving with the escape speed does not appear. Thus, the escape speed for the 1 000-kg spacecraft is the same as that for the 5 000-kg spacecraft. The only change in the kinetic energy is due to the mass, so the 1 000-kg spacecraft will require one fifth of the energy of the 5 000-kg spacecraft:

$$K = \tfrac{1}{5}(3.14 \times 10^{11}\ \text{J}) = 6.28 \times 10^{10}\ \text{J}$$

Table 13.3

Escape Speeds from the Surfaces of the Planets, Moon, and Sun

Planet	v_{esc} (km/s)
Mercury	4.3
Venus	10.3
Earth	11.2
Mars	5.0
Jupiter	60
Saturn	36
Uranus	22
Neptune	24
Pluto	1.1
Moon	2.3
Sun	618

Equations 13.21 and 13.22 can be applied to objects projected from any planet. That is, in general, the escape speed from the surface of any planet of mass M and radius R is

$$v_{esc} = \sqrt{\frac{2GM}{R}} \tag{13.23}$$

Escape speeds for the planets, the Moon, and the Sun are provided in Table 13.3. Note that the values vary from 1.1 km/s for Pluto to about 618 km/s for the Sun. These results, together with some ideas from the kinetic theory of gases (see Chapter 21), explain why some planets have atmospheres and others do not. As we shall see later, at a given temperature the average kinetic energy of a gas molecule depends only on the mass of the molecule. Lighter molecules, such as hydrogen and helium, have a higher average speed than heavier molecules at the same temperature. When the average speed of the lighter molecules is not much less than the escape speed of a planet, a significant fraction of them have a chance to escape.

This mechanism also explains why the Earth does not retain hydrogen molecules and helium atoms in its atmosphere but does retain heavier molecules, such as oxygen and nitrogen. On the other hand, the very large escape speed for Jupiter enables that planet to retain hydrogen, the primary constituent of its atmosphere.

Black Holes

In Example 11.7 we briefly described a rare event called a supernova—the catastrophic explosion of a very massive star. The material that remains in the central core of such an object continues to collapse, and the core's ultimate fate depends on its mass. If the core has a mass less than 1.4 times the mass of our Sun, it gradually cools down and ends its life as a white dwarf star. However, if the core's mass is greater than this, it may collapse further due to gravitational forces. What remains is a neutron star, discussed in Example 11.7, in which the mass of a star is compressed to a radius of about 10 km. (On Earth, a teaspoon of this material would weigh about 5 billion tons!)

An even more unusual star death may occur when the core has a mass greater than about three solar masses. The collapse may continue until the star becomes a very small object in space, commonly referred to as a **black hole.** In effect, black holes are remains of stars that have collapsed under their own gravitational force. If an object such as a spacecraft comes close to a black hole, it experiences an extremely strong gravitational force and is trapped forever.

The escape speed for a black hole is very high, due to the concentration of the mass of the star into a sphere of very small radius (see Eq. 13.23). If the escape speed exceeds the speed of light c, radiation from the object (such as visible light) cannot escape, and the object appears to be black; hence the origin of the terminology "black hole." The critical radius R_S at which the escape speed is c is called the **Schwarzschild radius** (Fig. 13.17). The imaginary surface of a sphere of this radius surrounding the black hole is called the **event horizon.** This is the limit of how close you can approach the black hole and hope to escape.

Although light from a black hole cannot escape, light from events taking place near the black hole should be visible. For example, it is possible for a binary star system to consist of one normal star and one black hole. Material surrounding the ordinary star can be pulled into the black hole, forming an **accretion disk** around the black hole, as suggested in Figure 13.18. Friction among particles in the accretion disk results in transformation of mechanical energy into internal energy. As a result, the orbital height of the material above the event horizon decreases and the temperature rises. This high-temperature material emits a large amount of radiation, extending well into the x-ray region of the electromagnetic spectrum. These x-rays are characteristic of a black hole. Several possible candidates for black holes have been identified by observation of these x-rays.

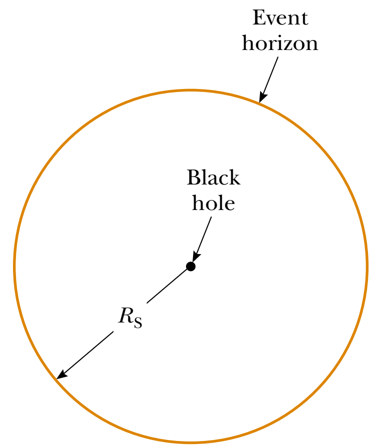


Figure 13.17 A black hole. The distance R_S equals the Schwarzschild radius. Any event occurring within the boundary of radius R_S, called the event horizon, is invisible to an outside observer.

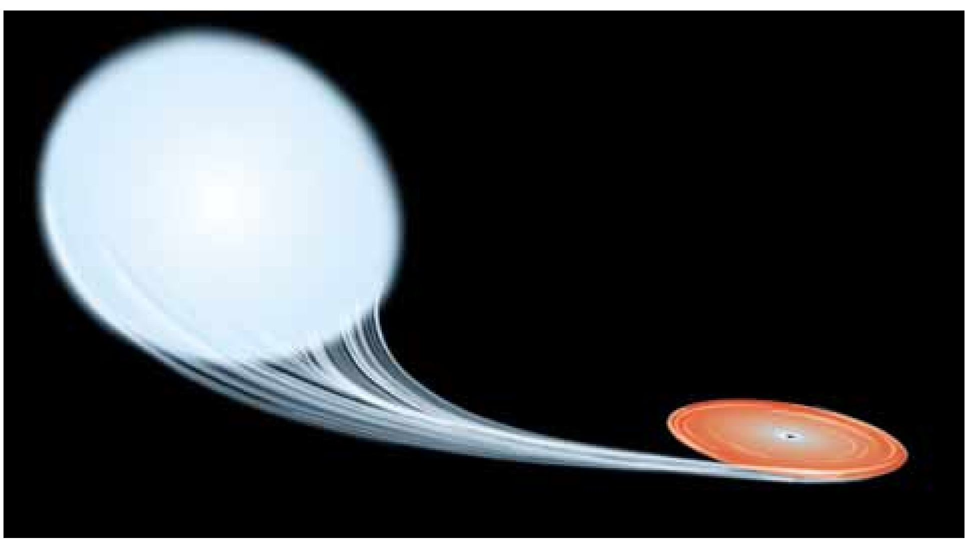
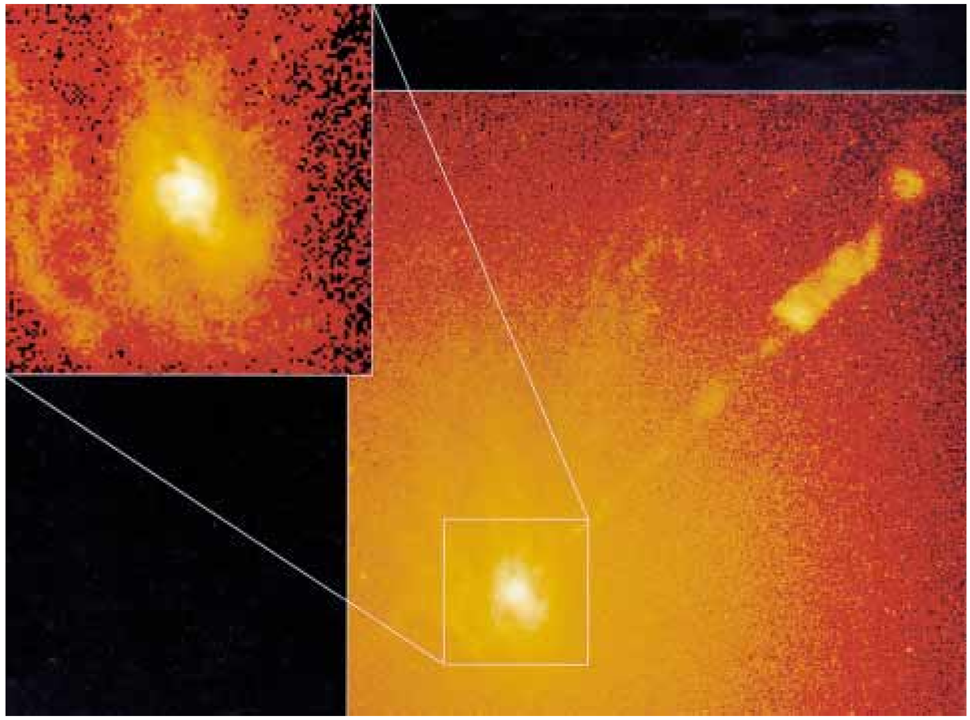


Figure 13.18 A binary star system consisting of an ordinary star on the left and a black hole on the right. Matter pulled from the ordinary star forms an accretion disk around the black hole, in which matter is raised to very high temperatures, resulting in the emission of x-rays.



H. Ford et al. & NASA

Figure 13.19 Hubble Space Telescope images of the galaxy M87. The inset shows the center of the galaxy. The wider view shows a jet of material moving away from the center of the galaxy toward the upper right of the figure at about one tenth of the speed of light. Such jets are believed to be evidence of a supermassive black hole at the galaxy center.

There is also evidence that supermassive black holes exist at the centers of galaxies, with masses very much larger than the Sun. (There is strong evidence of a supermassive black hole of mass 2–3 million solar masses at the center of our galaxy.) Theoretical models for these bizarre objects predict that jets of material should be evident along the rotation axis of the black hole. Figure 13.19 shows a Hubble Space Telescope photograph of galaxy M87. The jet of material coming from this galaxy is believed to be evidence for a supermassive black hole at the center of the galaxy.

SUMMARY

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

Newton's law of universal gravitation states that the gravitational force of attraction between any two particles of masses m_1 and m_2 separated by a distance r has the magnitude

$$F_g = G\frac{m_1 m_2}{r^2} \qquad (13.1)$$

where $G = 6.673 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$ is the **universal gravitational constant.** This equation enables us to calculate the force of attraction between masses under a wide variety of circumstances.

An object at a distance h above the Earth's surface experiences a gravitational force of magnitude mg, where g is the free-fall acceleration at that elevation:

$$g = \frac{GM_E}{r^2} = \frac{GM_E}{(R_E + h)^2} \qquad (13.6)$$

In this expression, M_E is the mass of the Earth and R_E is its radius. Thus, the weight of an object decreases as the object moves away from the Earth's surface.

Kepler's laws of planetary motion state that

1. All planets move in elliptical orbits with the Sun at one focus.
2. The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals.
3. The square of the orbital period of any planet is proportional to the cube of the semimajor axis of the elliptical orbit.

Kepler's third law can be expressed as

$$T^2 = \left(\frac{4\pi^2}{GM_S}\right)a^3 \tag{13.8}$$

where M_S is the mass of the Sun and a is the semimajor axis. For a circular orbit, a can be replaced in Equation 13.8 by the radius r. Most planets have nearly circular orbits around the Sun.

The **gravitational field** at a point in space is defined as the gravitational force experienced by any test particle located at that point divided by the mass of the test particle:

$$\mathbf{g} \equiv \frac{\mathbf{F}_g}{m} \tag{13.9}$$

The gravitational force is conservative, and therefore a potential energy function can be defined for a system of two objects interacting gravitationally. The **gravitational potential energy** associated with two particles separated by a distance r is

$$U = -\frac{Gm_1m_2}{r} \tag{13.14}$$

where U is taken to be zero as $r \rightarrow \infty$. The total potential energy for a system of particles is the sum of energies for all pairs of particles, with each pair represented by a term of the form given by Equation 13.14.

If an isolated system consists of an object of mass m moving with a speed v in the vicinity of a massive object of mass M, the total energy E of the system is the sum of the kinetic and potential energies:

$$E = \tfrac{1}{2}mv^2 - \frac{GMm}{r} \tag{13.16}$$

The total energy is a constant of the motion. If the object moves in an elliptical orbit of semimajor axis a around the massive object and if $M \gg m$, the total energy of the system is

$$E = -\frac{GMm}{2a} \tag{13.19}$$

For a circular orbit, this same equation applies with $a = r$. The total energy is negative for any bound system.

The **escape speed** for an object projected from the surface of a planet of mass M and radius R is

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} \tag{13.23}$$

QUESTIONS

1. If the gravitational force on an object is directly proportional to its mass, why don't objects with large masses fall with greater acceleration than small ones?

2. The gravitational force exerted by the Sun on you is downward into the Earth at night, and upward into the sky during the day. If you had a sensitive enough bathroom scale, would you expect to weigh more at night than during the day? Note also that you are farther away from the Sun at night than during the day. Would you expect to weigh less?

3. Use Kepler's second law to convince yourself that the Earth must move faster in its orbit during December, when it is closest to the Sun, than during June, when it is farthest from the Sun.

4. The gravitational force that the Sun exerts on the Moon is about twice as great as the gravitational force that the Earth exerts on the Moon. Why doesn't the Sun pull the Moon away from the Earth during a total eclipse of the Sun?

5. A satellite in orbit is not truly traveling through a vacuum. It is moving through very, very thin air. Does the resulting air friction cause the satellite to slow down?

6. How would you explain the fact that Jupiter and Saturn have periods much greater than one year?

7. If a system consists of five particles, how many terms appear in the expression for the total potential energy? How many terms appear if the system consists of N particles?

8. Does the escape speed of a rocket depend on its mass? Explain.

9. Compare the energies required to reach the Moon for a 10^5-kg spacecraft and a 10^3-kg satellite.

10. Explain why it takes more fuel for a spacecraft to travel from the Earth to the Moon than for the return trip. Estimate the difference.

11. A particular set of directions forms the *celestial equator*. If you live at 40° north latitude, these directions lie in an arc across your southern sky, including horizontally east, horizontally west, and south at 50° above the horizontal. In order to enjoy satellite TV, you need to install a dish with an unobstructed view to a particular point on the celestial equator. Why is this requirement so specific?

12. Why don't we put a geosynchronous weather satellite in orbit around the 45th parallel? Wouldn't this be more useful in the United States than one in orbit around the equator?

13. Is the absolute value of the potential energy associated with the Earth–Moon system greater than, less than, or equal to the kinetic energy of the Moon relative to the Earth?

14. Explain why no work is done on a planet as it moves in a circular orbit around the Sun, even though a gravitational force is acting on the planet. What is the net work done on a planet during each revolution as it moves around the Sun in an elliptical orbit?

15. Explain why the force exerted on a particle by a uniform sphere must be directed toward the center of the sphere. Would this be the case if the mass distribution of the sphere were not spherically symmetric?

16. At what position in its elliptical orbit is the speed of a planet a maximum? At what position is the speed a minimum?

17. If you are given the mass and radius of planet X, how would you calculate the free-fall acceleration on the surface of this planet?

18. If a hole could be dug to the center of the Earth, would the force on an object of mass m still obey Equation 13.1 there? What do you think the force on m would be at the center of the Earth?

19. In his 1798 experiment, Cavendish was said to have "weighed the Earth." Explain this statement.

20. The *Voyager* spacecraft was accelerated toward escape speed from the Sun by Jupiter's gravitational force exerted on the spacecraft. How is this possible?

21. How would you find the mass of the Moon?

22. The *Apollo 13* spacecraft developed trouble in the oxygen system about halfway to the Moon. Why did the mission continue on around the Moon, and then return home, rather than immediately turn back to Earth?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 13.1 Newton's Law of Universal Gravitation

Problem 17 in Chapter 1 can also be assigned with this section.

1. Determine the order of magnitude of the gravitational force that you exert on another person 2 m away. In your solution state the quantities you measure or estimate and their values.

2. Two ocean liners, each with a mass of 40 000 metric tons, are moving on parallel courses, 100 m apart. What is the magnitude of the acceleration of one of the liners toward the other due to their mutual gravitational attraction? Treat the ships as particles.

3. A 200-kg object and a 500-kg object are separated by 0.400 m. (a) Find the net gravitational force exerted by these objects on a 50.0-kg object placed midway between them. (b) At what position (other than an infinitely remote one) can the 50.0-kg object be placed so as to experience a net force of zero?

4. Two objects attract each other with a gravitational force of magnitude 1.00×10^{-8} N when separated by 20.0 cm. If the total mass of the two objects is 5.00 kg, what is the mass of each?

5. Three uniform spheres of mass 2.00 kg, 4.00 kg, and 6.00 kg are placed at the corners of a right triangle as in Figure P13.5. Calculate the resultant gravitational force on the 4.00-kg object, assuming the spheres are isolated from the rest of the Universe.

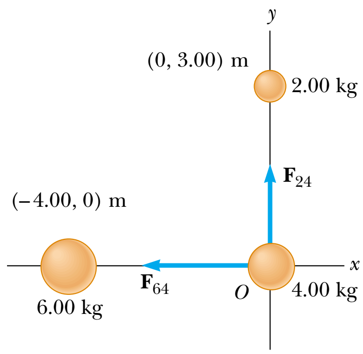


Figure P13.5

6. During a solar eclipse, the Moon, Earth, and Sun all lie on the same line, with the Moon between the Earth and the Sun. (a) What force is exerted by the Sun on the Moon? (b) What force is exerted by the Earth on the Moon? (c) What force is exerted by the Sun on the Earth?

Section 13.2 Measuring the Gravitational Constant

7. In introductory physics laboratories, a typical Cavendish balance for measuring the gravitational constant G uses lead spheres with masses of 1.50 kg and 15.0 g whose centers are separated by about 4.50 cm. Calculate the gravitational force between these spheres, treating each as a particle located at the center of the sphere.

8. A student proposes to measure the gravitational constant G by suspending two spherical objects from the ceiling of a tall cathedral and measuring the deflection of the cables from the vertical. Draw a free-body diagram of one of the objects. If two 100.0-kg objects are suspended at the lower ends of cables 45.00 m long and the cables are attached to the ceiling 1.000 m apart, what is the separation of the objects?

Section 13.3 Free-Fall Acceleration and the Gravitational Force

9. When a falling meteoroid is at a distance above the Earth's surface of 3.00 times the Earth's radius, what is its acceleration due to the Earth's gravitation?

10. The free-fall acceleration on the surface of the Moon is about one sixth of that on the surface of the Earth. If the radius of the Moon is about $0.250R_E$, find the ratio of their average densities, $\rho_{\text{Moon}}/\rho_{\text{Earth}}$.

11. On the way to the Moon the *Apollo* astronauts reached a point where the Moon's gravitational pull became stronger than the Earth's. (a) Determine the distance of this point from the center of the Earth. (b) What is the acceleration due to the Earth's gravitation at this point?

Section 13.4 Kepler's Laws and the Motion of Planets

12. The center-to-center distance between Earth and Moon is 384 400 km. The Moon completes an orbit in 27.3 days. (a) Determine the Moon's orbital speed. (b) If gravity were switched off, the Moon would move along a straight line tangent to its orbit, as described by Newton's first law. In its actual orbit in 1.00 s, how far does the Moon fall below the tangent line and toward the Earth?

13. Plaskett's binary system consists of two stars that revolve in a circular orbit about a center of mass midway between them. This means that the masses of the two stars are equal (Fig. P13.13). Assume the orbital speed of each star is 220 km/s and the orbital period of each is 14.4 days. Find the mass M of each star. (For comparison, the mass of our Sun is 1.99×10^{30} kg.)

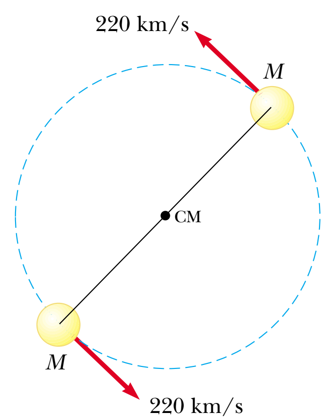


Figure P13.13

14. A particle of mass m moves along a straight line with constant speed in the x direction, a distance b from the x axis (Fig. P13.14). Show that Kepler's second law is satisfied by showing that the two shaded triangles in the figure have the same area when $t_4 - t_3 = t_2 - t_1$.

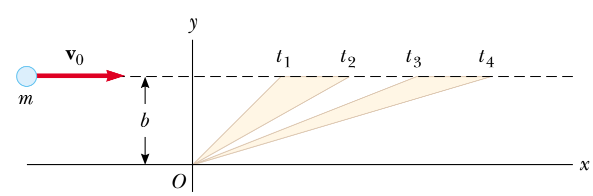


Figure P13.14

15. Io, a moon of Jupiter, has an orbital period of 1.77 days and an orbital radius of 4.22×10^5 km. From these data, determine the mass of Jupiter.

16. The *Explorer VIII* satellite, placed into orbit November 3, 1960, to investigate the ionosphere, had the following orbit parameters: perigee, 459 km; apogee, 2 289 km (both distances above the Earth's surface); period, 112.7 min. Find the ratio v_p/v_a of the speed at perigee to that at apogee.

17. Comet Halley (Figure P13.17) approaches the Sun to within 0.570 AU, and its orbital period is 75.6 years. (AU is the symbol for astronomical unit, where 1 AU = 1.50×10^{11} m is the mean Earth–Sun distance.) How far from the Sun will Halley's comet travel before it starts its return journey?

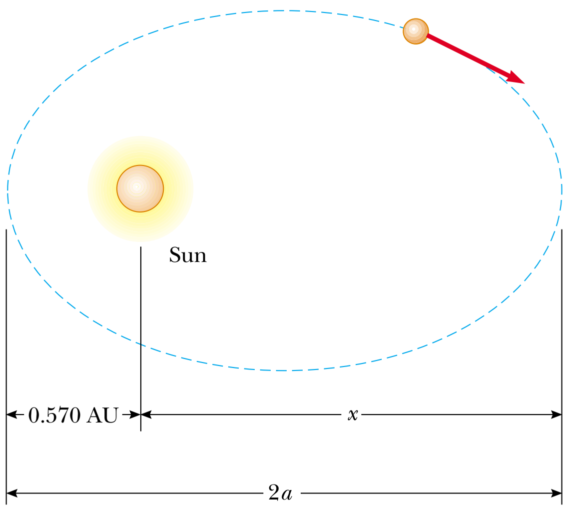


Figure P13.17

18. Two planets X and Y travel counterclockwise in circular orbits about a star as in Figure P13.18. The radii of their orbits are in the ratio 3 : 1. At some time, they are aligned as in Figure P13.18a, making a straight line with the star. During the next five years, the angular displacement of planet X is 90.0°, as in Figure P13.18b. Where is planet Y at this time?

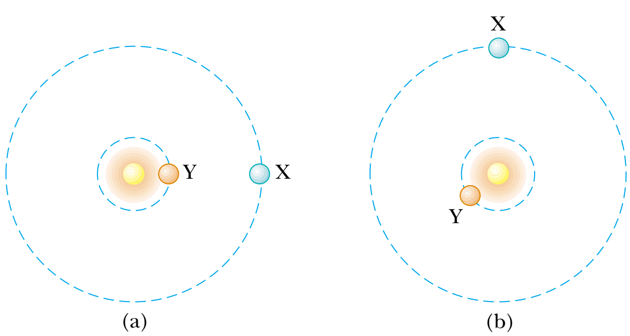


Figure P13.18

19. A synchronous satellite, which always remains above the same point on a planet's equator, is put in orbit around Jupiter to study the famous red spot. Jupiter rotates about its axis once every 9.84 h. Use the data of Table 13.2 to find the altitude of the satellite.

20. Neutron stars are extremely dense objects that are formed from the remnants of supernova explosions. Many rotate very rapidly. Suppose that the mass of a certain spherical neutron star is twice the mass of the Sun and its radius is 10.0 km. Determine the greatest possible angular speed it can have so that the matter at the surface of the star on its equator is just held in orbit by the gravitational force.

21. Suppose the Sun's gravity were switched off. The planets would leave their nearly circular orbits and fly away in straight lines, as described by Newton's first law. Would Mercury ever be farther from the Sun than Pluto? If so, find how long it would take for Mercury to achieve this passage. If not, give a convincing argument that Pluto is always farther from the Sun.

22. As thermonuclear fusion proceeds in its core, the Sun loses mass at a rate of 3.64×10^{9} kg/s. During the 5 000-yr period of recorded history, by how much has the length of the year changed due to the loss of mass from the Sun? *Suggestions:* Assume the Earth's orbit is circular. No external torque acts on the Earth–Sun system, so its angular momentum is conserved. If x is small compared to 1, then $(1 + x)^n$ is nearly equal to $1 + nx$.

Section 13.5 The Gravitational Field

23. Three objects of equal mass are located at three corners of a square of edge length ℓ as in Figure P13.23. Find the gravitational field at the fourth corner due to these objects.

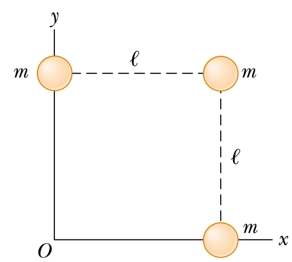


Figure P13.23

24. A spacecraft in the shape of a long cylinder has a length of 100 m, and its mass with occupants is 1 000 kg. It has strayed too close to a black hole having a mass 100 times that of the Sun (Fig. P13.24). The nose of the spacecraft points toward the black hole, and the distance between the nose and the center of the black hole is 10.0 km. (a) Determine the total force on the spacecraft. (b) What is the difference in the gravitational fields acting on the occupants in the nose of the ship and on those in the rear of the ship, farthest from the black hole? This difference in accelerations grows rapidly as the ship approaches the black hole. It puts the body of the ship under extreme tension and eventually tears it apart.

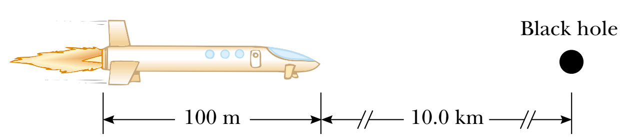


Figure P13.24

25. Compute the magnitude and direction of the gravitational field at a point P on the perpendicular bisector of the line joining two objects of equal mass separated by a distance $2a$ as shown in Figure P13.25.

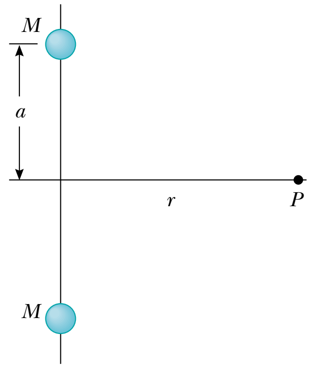


Figure P13.25

Section 13.6 Gravitational Potential Energy

> Assume $U = 0$ at $r = \infty$.

26. A satellite of the Earth has a mass of 100 kg and is at an altitude of 2.00×10^6 m. (a) What is the potential energy of the satellite–Earth system? (b) What is the magnitude of the gravitational force exerted by the Earth on the satellite? (c) **What If?** What force does the satellite exert on the Earth?

27. How much energy is required to move a 1 000-kg object from the Earth's surface to an altitude twice the Earth's radius?

28. At the Earth's surface a projectile is launched straight up at a speed of 10.0 km/s. To what height will it rise? Ignore air resistance and the rotation of the Earth.

29. After our Sun exhausts its nuclear fuel, its ultimate fate may be to collapse to a *white dwarf* state, in which it has approximately the same mass as it has now, but a radius equal to the radius of the Earth. Calculate (a) the average density of the white dwarf, (b) the free-fall acceleration, and (c) the gravitational potential energy of a 1.00-kg object at its surface.

30. How much work is done by the Moon's gravitational field as a 1 000-kg meteor comes in from outer space and impacts on the Moon's surface?

31. A system consists of three particles, each of mass 5.00 g, located at the corners of an equilateral triangle with sides of 30.0 cm. (a) Calculate the potential energy of the system. (b) If the particles are released simultaneously, where will they collide?

32. An object is released from rest at an altitude h above the surface of the Earth. (a) Show that its speed at a distance r from the Earth's center, where $R_E \le r \le R_E + h$, is given by

$$v = \sqrt{2GM_E\left(\frac{1}{r} - \frac{1}{R_E + h}\right)}$$

(b) Assume the release altitude is 500 km. Perform the integral

$$\Delta t = \int_i^f dt = -\int_i^f \frac{dr}{v}$$

to find the time of fall as the object moves from the release point to the Earth's surface. The negative sign appears because the object is moving opposite to the radial direction, so its speed is $v = -dr/dt$. Perform the integral numerically.

Section 13.7 Energy Considerations in Planetary and Satellite Motion

33. A space probe is fired as a projectile from the Earth's surface with an initial speed of 2.00×10^4 m/s. What will its speed be when it is very far from the Earth? Ignore friction and the rotation of the Earth.

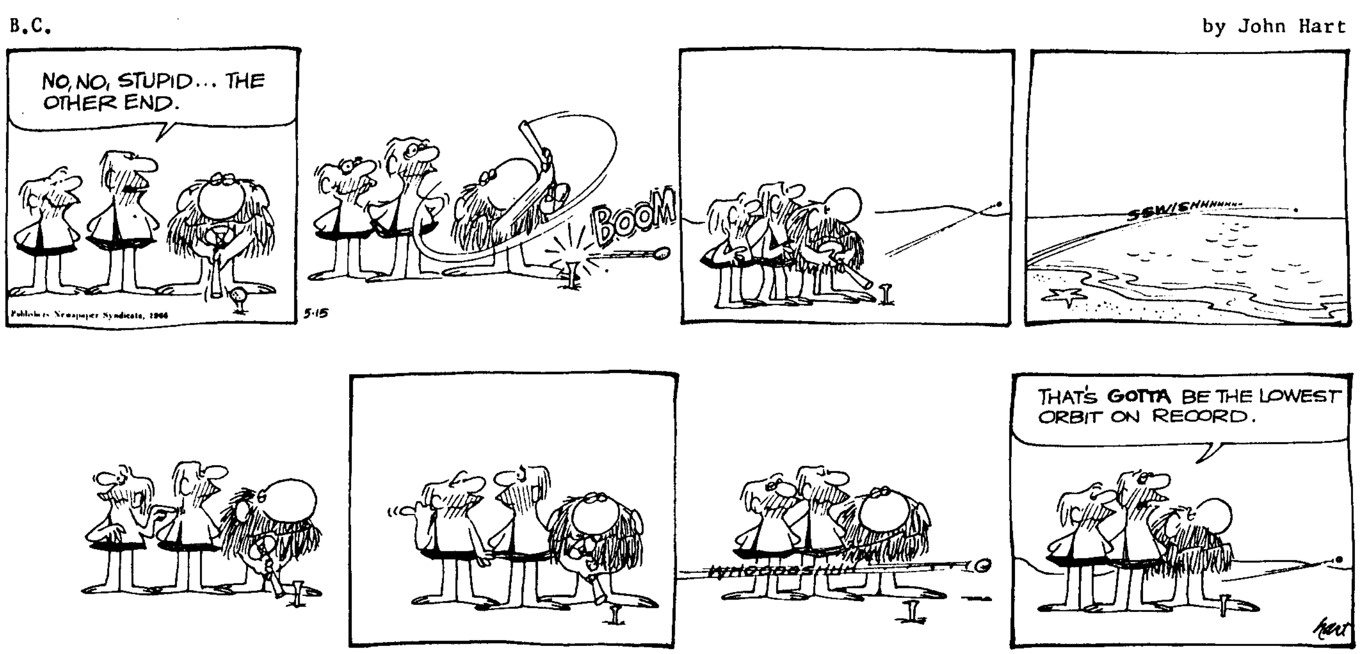


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Figure P13.35

34. (a) What is the minimum speed, relative to the Sun, necessary for a spacecraft to escape the solar system if it starts at the Earth's orbit? (b) *Voyager 1* achieved a maximum speed of 125 000 km/h on its way to photograph Jupiter. Beyond what distance from the Sun is this speed sufficient to escape the solar system?

35. A "treetop satellite" (Fig. P13.35) moves in a circular orbit just above the surface of a planet, assumed to offer no air resistance. Show that its orbital speed v and the escape speed from the planet are related by the expression

$$v_{\text{esc}} = \sqrt{2}\,v.$$

36. A 500-kg satellite is in a circular orbit at an altitude of 500 km above the Earth's surface. Because of air friction, the satellite eventually falls to the Earth's surface, where it hits the ground with a speed of 2.00 km/s. How much energy was transformed into internal energy by means of friction?

37. A satellite of mass 200 kg is placed in Earth orbit at a height of 200 km above the surface. (a) With a circular orbit, how long does the satellite take to complete one orbit? (b) What is the satellite's speed? (c) What is the minimum energy input necessary to place this satellite in orbit? Ignore air resistance but include the effect of the planet's daily rotation.

38. A satellite of mass m, originally on the surface of the Earth, is placed into Earth orbit at an altitude h. (a) With a circular orbit, how long does the satellite take to complete one orbit? (b) What is the satellite's speed? (c) What is the minimum energy input necessary to place this satellite in orbit? Ignore air resistance but include the effect of the planet's daily rotation. At what location on the Earth's surface and in what direction should the satellite be launched to minimize the required energy investment? Represent the mass and radius of the Earth as M_E and R_E.

39. A 1 000-kg satellite orbits the Earth at a constant altitude of 100 km. How much energy must be added to the system to move the satellite into a circular orbit with altitude 200 km?

40. The planet Uranus has a mass about 14 times the Earth's mass, and its radius is equal to about 3.7 Earth radii. (a) By setting up ratios with the corresponding Earth values, find the free-fall acceleration at the cloud tops of Uranus. (b) Ignoring the rotation of the planet, find the minimum escape speed from Uranus.

41. Determine the escape speed for a rocket on the far side of Ganymede, the largest of Jupiter's moons (Figure P13.41). The radius of Ganymede is 2.64×10^6 m, and its mass is 1.495×10^{23} kg. The mass of Jupiter is 1.90×10^{27} kg, and the distance between Jupiter and Ganymede is 1.071×10^9 m. Be sure to include the gravitational effect due to Jupiter, but you may ignore the motion of Jupiter and Ganymede as they revolve about their center of mass.

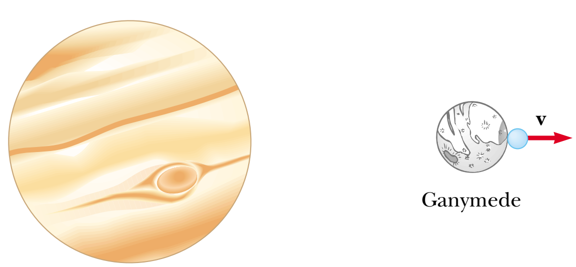

Figure P13.41

42. In Robert Heinlein's "The Moon is a Harsh Mistress," the colonial inhabitants of the Moon threaten to launch rocks down onto the Earth if they are not given independence (or at least representation). Assuming that a rail gun could launch a rock of mass m at twice the lunar escape speed, calculate the speed of the rock as it enters the Earth's atmosphere. (By *lunar escape speed* we mean the speed required to move infinitely far away from a stationary Moon alone in the Universe. Problem 61 in Chapter 30 describes a rail gun.)

43. An object is fired vertically upward from the surface of the Earth (of radius R_E) with an initial speed v_i that is comparable to but less than the escape speed v_{esc}. (a) Show that the object attains a maximum height h given by

$$h = \frac{R_E v_i^2}{v_{\text{esc}}^2 - v_i^2}$$

(b) A space vehicle is launched vertically upward from the Earth's surface with an initial speed of 8.76 km/s, which is less than the escape speed of 11.2 km/s. What maximum height does it attain? (c) A meteorite falls toward the Earth. It is essentially at rest with respect to the Earth when it is at a height of 2.51×10^7 m. With what speed does the meteorite strike the Earth? (d) **What If?** Assume that a baseball is tossed up with an initial speed that is very small compared to the escape speed. Show that the equation from part (a) is consistent with Equation 4.13.

44. Derive an expression for the work required to move an Earth satellite of mass m from a circular orbit of radius $2R_E$ to one of radius $3R_E$.

45. A comet of mass 1.20×10^{10} kg moves in an elliptical orbit around the Sun. Its distance from the Sun ranges between 0.500 AU and 50.0 AU. (a) What is the eccentricity of its orbit? (b) What is its period? (c) At aphelion what is the potential energy of the comet–Sun system? *Note:* 1 AU = one astronomical unit = the average distance from Sun to Earth = 1.496×10^{11} m.

46. A satellite moves around the Earth in a circular orbit of radius r. (a) What is the speed v_0 of the satellite? Suddenly, an explosion breaks the satellite into two pieces, with masses m and $4m$. Immediately after the explosion the smaller piece of mass m is stationary with respect to the Earth and falls directly toward the Earth. (b) What is the speed v_i of the larger piece immediately after the explosion? (c) Because of the increase in its speed, this larger piece now moves in a new elliptical orbit. Find its distance away from the center of the Earth when it reaches the other end of the ellipse.

Additional Problems

47. The Solar and Heliospheric Observatory (SOHO) spacecraft has a special orbit, chosen so that its view of the Sun is never eclipsed and it is always close enough to the Earth to transmit data easily. It moves in a near-circle around the Sun that is smaller than the Earth's circular orbit. Its period, however, is just equal to 1 yr. It is always located between the Earth and the Sun along the line joining them. Both objects exert gravitational forces on the observatory. Show that its distance from the Earth must be between 1.47×10^9 m and 1.48×10^9 m. In 1772 Joseph Louis Lagrange determined theoretically the special location allowing this orbit. The SOHO spacecraft took this position on February 14, 1996. *Suggestion:* Use data that are precise to four digits. The mass of the Earth is 5.983×10^{24} kg.

48. Let Δg_M represent the difference in the gravitational fields produced by the Moon at the points on the Earth's surface nearest to and farthest from the Moon. Find the fraction $\Delta g_M/g$, where g is the Earth's gravitational field. (This difference is responsible for the occurrence of the *lunar tides* on the Earth.)

49. **Review problem.** Two identical hard spheres, each of mass m and radius r, are released from rest in otherwise empty space with their centers separated by the distance R. They are allowed to collide under the influence of their gravitational attraction. (a) Show that the magnitude of the impulse received by each sphere before they make contact is given by $[Gm^3(1/2r - 1/R)]^{1/2}$. (b) **What If?** Find the magnitude of the impulse each receives if they collide elastically.

50. Two spheres having masses M and $2M$ and radii R and $3R$, respectively, are released from rest when the distance between their centers is $12R$. How fast will each sphere be moving when they collide? Assume that the two spheres interact only with each other.

51. In Larry Niven's science-fiction novel *Ringworld,* a rigid ring of material rotates about a star (Fig. P13.51). The tangential speed of the ring is 1.25×10^6 m/s, and its radius is 1.53×10^{11} m. (a) Show that the centripetal acceleration of the inhabitants is 10.2 m/s^2. (b) The inhabitants of this ring world live on the starlit inner surface of the ring. Each person experiences a normal contact force $\mathbf{n}$. Acting alone, this normal force would produce an inward acceleration of 9.90 m/s^2. Additionally, the star at the center of the ring exerts a gravitational force on the ring and its inhabitants. The difference between the total acceleration and the acceleration provided by the normal force is due to the gravitational attraction of the central star. Show that the mass of the star is approximately 10^{32} kg.

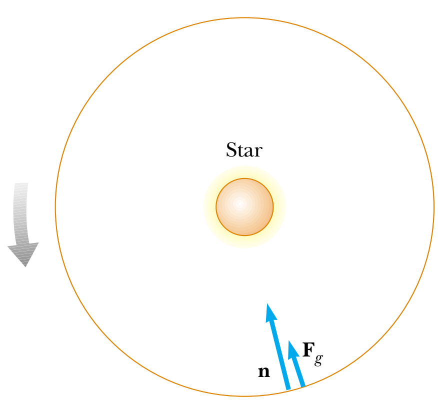

Figure P13.51

52. (a) Show that the rate of change of the free-fall acceleration with distance above the Earth's surface is

$$\frac{dg}{dr} = -\frac{2GM_E}{{R_E}^3}$$

This rate of change over distance is called a *gradient.* (b) If h is small in comparison to the radius of the Earth, show that the difference in free-fall acceleration between two points separated by vertical distance h is

$$|\Delta g| = \frac{2GM_E h}{{R_E}^3}$$

(c) Evaluate this difference for $h = 6.00$ m, a typical height for a two-story building.

53. A ring of matter is a familiar structure in planetary and stellar astronomy. Examples include Saturn's rings and a ring nebula. Consider a uniform ring of mass 2.36×10^{20} kg and radius 1.00×10^8 m. An object of mass 1 000 kg is placed at a point A on the axis of the ring, 2.00×10^8 m from the center of the ring (Figure P13.53). When the object is released, the attraction of the ring makes the object move along the axis toward the center of the ring (point B). (a) Calculate the gravitational potential energy of the object–ring system when the object is at A. (b) Calculate the gravitational potential energy of the system when the object is at B. (c) Calculate the speed of the object as it passes through B.

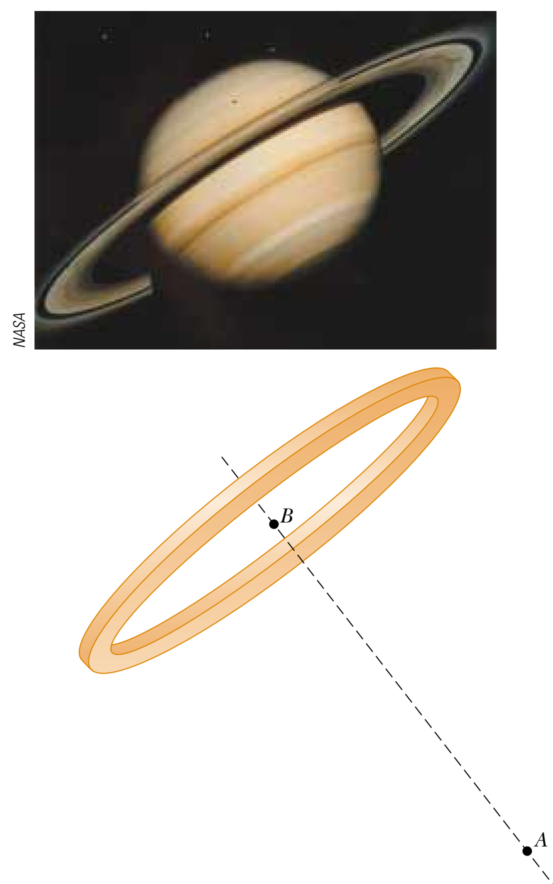

Figure P13.53

54. *Voyagers 1* and *2* surveyed the surface of Jupiter's moon Io and photographed active volcanoes spewing liquid sulfur to heights of 70 km above the surface of this moon. Find the speed with which the liquid sulfur left the volcano. Io's mass is 8.9×10^{22} kg, and its radius is 1 820 km.

55. As an astronaut, you observe a small planet to be spherical. After landing on the planet, you set off, walking always straight ahead, and find yourself returning to your spacecraft from the opposite side after completing a lap of 25.0 km. You hold a hammer and a falcon feather at a height of 1.40 m, release them, and observe that they fall together to the surface in 29.2 s. Determine the mass of the planet.

56. A certain quaternary star system consists of three stars, each of mass m, moving in the same circular orbit of radius r about a central star of mass M. The stars orbit in the same sense, and are positioned one third of a revolution apart from each other. Show that the period of each of the three stars is given by

$$T = 2\pi\sqrt{\frac{r^3}{g\left(M + m/\sqrt{3}\right)}}$$

57. Review problem. A cylindrical habitat in space 6.00 km in diameter and 30 km long has been proposed (by G. K. O'Neill, 1974). Such a habitat would have cities, land, and lakes on the inside surface and air and clouds in the center. This would all be held in place by rotation of the cylinder about its long axis. How fast would the cylinder have to rotate to imitate the Earth's gravitational field at the walls of the cylinder?

58. Newton's law of universal gravitation is valid for distances covering an enormous range, but it is thought to fail for very small distances, where the structure of space itself is uncertain. Far smaller than an atomic nucleus, this crossover distance is called the Planck length. It is determined by a combination of the constants G, c, and h, where c is the speed of light in vacuum and h is Planck's constant (introduced in Chapter 11) with units of angular momentum. (a) Use dimensional analysis to find a combination of these three universal constants that has units of length. (b) Determine the order of magnitude of the Planck length. You will need to consider noninteger powers of the constants.

59. Show that the escape speed from the surface of a planet of uniform density is directly proportional to the radius of the planet.

60. Many people assume that air resistance acting on a moving object will always make the object slow down. It can actually be responsible for making the object speed up. Consider a 100-kg Earth satellite in a circular orbit at an altitude of 200 km. A small force of air resistance makes the satellite drop into a circular orbit with an altitude of 100 km. (a) Calculate its initial speed. (b) Calculate its final speed in this process. (c) Calculate the initial energy of the satellite–Earth system. (d) Calculate the final energy of the system. (e) Show that the system has lost mechanical energy and find the amount of the loss due to friction. (f) What force makes the satellite's speed increase? You will find a free-body diagram useful in explaining your answer.

61. Two hypothetical planets of masses m_1 and m_2 and radii r_1 and r_2, respectively, are nearly at rest when they are an infinite distance apart. Because of their gravitational attraction, they head toward each other on a collision course. (a) When their center-to-center separation is d, find expressions for the speed of each planet and for their relative speed. (b) Find the kinetic energy of each planet just before they collide, if $m_1 = 2.00 \times 10^{24}$ kg, $m_2 = 8.00 \times 10^{24}$ kg, $r_1 = 3.00 \times 10^6$ m, and $r_2 = 5.00 \times 10^6$ m. (*Note:* Both energy and momentum of the system are conserved.)

62. The maximum distance from the Earth to the Sun (at our aphelion) is 1.521×10^{11} m, and the distance of closest approach (at perihelion) is 1.471×10^{11} m. If the Earth's orbital speed at perihelion is 3.027×10^4 m/s, determine (a) the Earth's orbital speed at aphelion, (b) the kinetic and potential energies of the Earth–Sun system at perihelion, and (c) the kinetic and potential energies at aphelion. Is the total energy constant? (Ignore the effect of the Moon and other planets.)

63. (a) Determine the amount of work (in joules) that must be done on a 100-kg payload to elevate it to a height of 1 000 km above the Earth's surface. (b) Determine the amount of additional work that is required to put the payload into circular orbit at this elevation.

64. X-ray pulses from Cygnus X-1, a celestial x-ray source, have been recorded during high-altitude rocket flights. The signals can be interpreted as originating when a blob of ionized matter orbits a black hole with a period of 5.0 ms. If the blob were in a circular orbit about a black hole whose mass is $20M_{\text{Sun}}$, what is the orbit radius?

65. Studies of the relationship of the Sun to its galaxy—the Milky Way—have revealed that the Sun is located near the outer edge of the galactic disk, about 30 000 lightyears from the center. The Sun has an orbital speed of approximately 250 km/s around the galactic center. (a) What is the period of the Sun's galactic motion? (b) What is the order of magnitude of the mass of the Milky Way galaxy? Suppose that the galaxy is made mostly of stars of which the Sun is typical. What is the order of magnitude of the number of stars in the Milky Way?

66. The oldest artificial satellite in orbit is *Vanguard I*, launched March 3, 1958. Its mass is 1.60 kg. In its initial orbit, its minimum distance from the center of the Earth was 7.02 Mm, and its speed at this perigee point was 8.23 km/s. (a) Find the total energy of the satellite–Earth system. (b) Find the magnitude of the angular momentum of the satellite. (c) Find its speed at apogee and its maximum (apogee) distance from the center of the Earth. (d) Find the semimajor axis of its orbit. (e) Determine its period.

67. Astronomers detect a distant meteoroid moving along a straight line that, if extended, would pass at a distance $3R_E$ from the center of the Earth, where R_E is the radius of the Earth. What minimum speed must the meteoroid have if the Earth's gravitation is not to deflect the meteoroid to make it strike the Earth?

68. A spherical planet has uniform density ρ. Show that the minimum period for a satellite in orbit around it is

$$T_{\text{min}} = \sqrt{\frac{3\pi}{G\rho}}$$

independent of the radius of the planet.

69. Two stars of masses M and m, separated by a distance d, revolve in circular orbits about their center of mass (Fig. P13.69). Show that each star has a period given by

$$T^2 = \frac{4\pi^2 d^3}{G(M + m)}$$

Proceed as follows: Apply Newton's second law to each star. Note that the center-of-mass condition requires that $Mr_2 = mr_1$, where $r_1 + r_2 = d$.

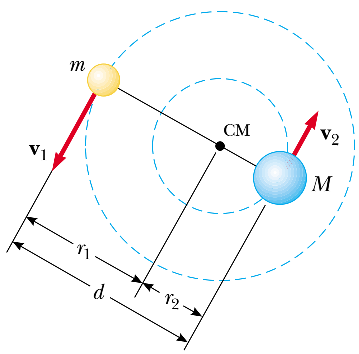

Figure P13.69

70. (a) A 5.00-kg object is released 1.20×10^7 m from the center of the Earth. It moves with what acceleration relative to the Earth? (b) **What If?** A 2.00×10^{24} kg object is released 1.20×10^7 m from the center of the Earth. It moves with what acceleration relative to the Earth? Assume that the objects behave as pairs of particles, isolated from the rest of the Universe.

71. The acceleration of an object moving in the gravitational field of the Earth is

$$\mathbf{a} = -\frac{GM_E\mathbf{r}}{r^3}$$

where $\mathbf{r}$ is the position vector directed from the center of the Earth toward the object. Choosing the origin at the center of the Earth and assuming that the small object is moving in the xy plane, we find that the rectangular (Cartesian) components of its acceleration are

$$a_x = -\frac{GM_E x}{(x^2 + y^2)^{3/2}} \qquad a_y = -\frac{GM_E y}{(x^2 + y^2)^{3/2}}$$

Use a computer to set up and carry out a numerical prediction of the motion of the object, according to Euler's method. Assume the initial position of the object is $x = 0$ and $y = 2R_E$, where R_E is the radius of the Earth. Give the object an initial velocity of 5 000 m/s in the x direction. The time increment should be made as small as practical. Try 5 s. Plot the x and y coordinates of the object as time goes on. Does the object hit the Earth? Vary the initial velocity until you find a circular orbit.

Answers to Quick Quizzes

13.1 (d). The gravitational force exerted by the Earth on the Moon provides a net force that causes the Moon's centripetal acceleration.

13.2 (e). The gravitational force follows an inverse-square behavior, so doubling the distance causes the force to be one fourth as large.

13.3 (c). An object in orbit is simply falling while it moves around the Earth. The acceleration of the object is that due to gravity. Because the object was launched from a very tall mountain, the value for g is slightly less than that at the surface.

13.4 (a). Kepler's third law (Eq. 13.8), which applies to all the planets, tells us that the period of a planet is proportional to $a^{3/2}$. Because Pluto is farther from the Sun than the Earth, it has a longer period. The Sun's gravitational field is much weaker at Pluto than it is at the Earth. Thus, this planet experiences much less centripetal acceleration than the Earth does, and it has a correspondingly longer period.

13.5 (a). From Kepler's third law and the given period, the major axis of the asteroid can be calculated. It is found to be 1.2×10^{11} m. Because this is smaller than the Earth–Sun distance, the asteroid cannot possibly collide with the Earth.

13.6 (b). From conservation of angular momentum, $mv_pr_p = mv_ar_a$, so that $v_p = (r_a/r_p)v_a = (4D/D)v_a = 4v_a$.

13.7 (a) Perihelion. Because of conservation of angular momentum, the speed of the comet is highest at its closest position to the Sun. (b) Aphelion. The potential energy of the comet–Sun system is highest when the comet is at its farthest distance from the Sun. (c) Perihelion. The kinetic energy is highest at the point at which the speed of the comet is highest. (d) All points. The total energy of the system is the same regardless of where the comet is in its orbit.